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AUTOMATIC CANNON TECHNOLOGY (ACT) 30MM SELF-POWERED FIRING FIXT--ETC(U)
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UNCLASSIFIED RIA-R-TR-77-011 NL

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AUTOMATIC CANNON TECHNOLOGY (ACT) 30MM SELF-POWERED FIRING FIXTURE

APRIL 77

STATUS REPORT

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GENERAL THOMAS J. RODMAN LABORATORY
ROCK ISLAND ARSENAL
ROCK ISLAND, ILLINOIS 61201

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This report outlines the concept, progress and test results of a high impulse 30 MM firing fixture.

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INTRODUCTION

This report documents work done on the Electro-Mechanical-Constant Recoil study performed under DH96 and AH78 at Rock Island Arsenal, Rock Ialand, IL, 61201.

CONCEPT

A self-powered, high impulse generator was to be developed to interface with an analog logic system to enable test data be obtained with respect to a constant recoil system. The telescoped cased consolidated 30MM AMCAWS round was selected due to its availability and high impulse levels (150 lb/sec.) Internal reports relating to constant recoil (firing out of battery) were the starting point of this program. A self-powered version was decided upon since an externally powered weapon is being used for a recoil force reduction program underway at *Minneapolis-Honeywell of St. Louis Park*, Minnesota.

PROGRESS SUMMARY

The major objective of this effort was to design a self-powered firing fixture capable of handling medium to high impulses at varying rates of fire. The initial criteria was 425 SPM and 150 lb/sec impulse. The "ideal" minimum recoil force is obtained by accelerating the recoiling masses in the counter recoil (or forward) direction, up to a velocity that will cancel one-half of the round impulse of 150 lb/sec. The recoil/counterrecoil stroke of the main mass is 5 inches while the slide travels an additional 9 1/2 inches or a total of 14 1/2 inches. This extra slide travel is for feeding clearances and chamber locking. An analysis of the operating principle is in the Appendix.

TEST DATA

Single round firings utilizing a set of AMCAWS 30 recoil buffers were performed to determine proper gas orifice size in the barrel which is .156 in. and required slide spring energies to obtain reliable sear-up, which turned out to be 3500 in/lb. The slide recoil stroke of 9 1/2 in. took 33ms which is good. The counterrecoil stroke of the slide required 95ms which is too slow. Increasing spring energy further is detrimental to the recoil stroke (See Problem Solutions #7).

Results of 18 Single Round Firings.

1. Firing pin solenoid is inadequate due to low energy and high pull-in time of 73ms plus drop-out time of 137ms. The cycle time of the solenoid alone is 210ms = 286 cyc/min.
2. Bolt extension and stud rollers were too soft.
3. Feed space between bolt and barrel face was too small.
4. Firing pin assembly was too heavy increasing action time.
5. Piston configuration is satisfactory.
6. Gas port of .156 dia. (3 plcs) in barrel at 44" is satisfactory.
7. Drive springs for slide require 3500 in/lb energy.
8. Baseplate must be stiffened to assist sear operation.
9. Gun cycle time is:

Recoil stroke	35ms
Feed time	30ms
Counterrecoil stroke	95ms
Cycle time	160ms

A firing rate of 375 SPM results.

10. The firing solenoid + mechanism + round action times must be concurrent with the counterrecoil stroke time which creates logic lead-time problems.

11. The main sear required excessive force to release it requiring additional analysis.

12. Installation and removal of the slide springs is extremely difficult.

13. Main mass charger pinion skips teeth due to rack bowing under the 1200 lb. main mass buffers.

14. Loss of head space after each firing occurred due to lack of adequate barrel to receiver locking method.

15. Alignment of gas cylinder, to center guide, to piston, to slide, is critical.

<u>PROBLEM SOLUTIONS</u>
(Refer to test results above)

1. Solenoid and firing mechanism reworking reduced pull-in time to 20ms and drop-out time to 100ms for cycle time of 120ms which was still too long, therefore a redesigned firing pin mechanism and actuating device were designed and fabricated. See drawing 76F40268 for new design. Parts have been fabricated and assembled.

2. The bolt extension and stud rollers were hardened.

3. Space was increased by adding ¼ inch to recoil motion of slide there by having a 9 3/4 inch slide motion relative to the receiver.

4. See 1 above.

5. N/A

6. N/A

7. Drive springs for the slide are difficult to install and remove due to

the large forces involved. Since the reduction of spring load increases slide travel time which is presently too great an assembly procedure to offset this problem was devised. See II E of assembly procedures.

8. A new Firing Table was fabricated that incorporated all the functional needs of the original base plate.

9. N/A

10. See 1 above.

11. A new main-sear mounted in the right rear support plate is actuated by an air cylinder.

12. See 7. above.

13. The pinion and rack no longer charge against the 1200 lb main mass buffer. A separate main charger was designed.

14. A positive locking device must still be designed.

15. No analysis has been undertaken on this problem.

APPENDIX

AMC 30 ACT ASSEMBLY PROCEDURE

I. Firing Table Components Assembly

A. Crank Sub-Assembly (73B40299)

1. Pin crank, handle, bolt, and pinion shaft (73B40235) together.

2. Locate latch (75B40188) (large opening up) in lock (75B40189) and insert pinion shaft.

3. Place pinion (73C40242) over shaft and pin in place.

B. Slide Charger Sub-Assy

1. Locate charger block (75C40187) on table and secure with four bolts.

2. Insert spring (73B40238) in block.

3. Compress spring with shaft of crank sub-assy and fasten in place with two bolts. (See A for Crank Sub-Assy)

C. Rear Support Assembly

1. Locate brace (75C40182) on firing table and bolt in position.

2. Assemble left and right rear supports to brace. Assure 6.00 inch space between rail guides.

3. Assemble pivot block (75B40119), main sear (75C40120), leaf spring (75B40118), and retaining plate (75C40121), in that order to right rear support.

4. Holding main sear retracted, locate stud (76B40232) and insert cross pin (76B40230). Allow sear to return into sideplate which retains pin in position.

AFTER II COMPLETE THE FOLLOWING STEPS 5 & 6

5. With spacer plate in position, screw air cylinder on stud (4 above) until the main sear protrudes into plate mid-groove 1/8 inch.

6. Continue rotation until air inlet is in up position, then secure cylinder to side plate with 2 bolts.

D. Front Support Assy

1. Locate front spacer (76B40229), support (73D40225), and straps (76B40231) in that order on table and bolt down.

II. Weapon Assembly

A. Chamber - Bolt - Firing Pin Sub Assy (76D40270)

1. Start with chamber.

2. Insert bolt (75B40238) into chamber.

3. Assemble firing pin (76A40240), tube (76B40239), and retaining pin.

4. Insert spring (75A40183) and then 3 above into bolt.

5. Place backplate sub-assy (73C40297) on bolt.

6. Insert bolt extension (76C40237) in backplate.

7. Install crosspin (73B40183) thru bolt and tube holding it in position with spring (73B40181-C).

8. Compress spring with base (73B40186) and insert retaining pin thru tube and plug.

B. Barrel Sub-Assy

1. Locate headspace nut (73B40138) on barrel.

2. Place central guide (73C40208) and nut (73B40209) on barrel (DO NOT TIGHTEN).

3. Screw plug (73B40212) with seal (73B40207) into gas cylinder.

4. Slip gas cylinder and nut (73B40211) onto barrel (DO NOT TIGHTEN).

5. Insert gas piston (73D40147) into cylinder.

C. Barrel-Receiver Group Sub-Assy

1. Install receiver upside down in rear support (Stud slot up).

2. Screw barrel 3 inches into receiver.

3. Install rear slide guide (73C40144).

4. Insert chamber sub-assy into receiver screwing in backplate until it is flush with the rear of the receiver.

5. Screw in the backplate alignment screw (73B40177).

6. Rotate bolt so longitudinal slot lines up with the receiver stud slot.

7. Screw on housing (75D40138) hand tight.

8. Install stud (73B40159) and rollers (73B40158) (thin roller first).

9. Assemble round stop pawl (73B40161).

10. Insert headspace gage and lock chamber in forward position (gage set at 6.305).

11. Screw in barrel tightly and secure with head space nut.

12. Remove headspace gage.

13. Orientate central guide with holes up.

14. Slip on two spring housings (73C40215).

15. Lay slide sub-assy (See D Below) against receiver locating cam path over the stud roller.

16. Connect slide to rear slide guide and secure with pip pin.

17. Connect slide to piston with retaining pin.

D. Slide Sub-Assembly

1. Assemble slide safety using lock (75C40162), torsion spring (75B40161), and shoulder stud (75B40160) to the slide.

2. Insert pip pin thru sear into slide.

3. Install buffer lug (73B40142) and spring and pin in place.

4. Install sear assembly (73B40298) in the slide.

5. Locate sear holder (77C40003) on slide to compress sear assembly.

E. Slide Spring Installation

1. Turn barrel-receiver assy over in rear supports.

2. Assemble receiver strap (77B40001) to back of rear supports.

3. Clamp slide support (77C40002) 2" forward of rear support.

4. Engage pinion with slide rack and crank slide forward until rear slide guide pip pin is accessible.

5. Disengage rear slide guide and while holding receiver against 2. above continue to crank the slide forward until the pinion and rack are almost disengaged.

6. Remove slide safety pip pin.

7. Assemble drive springs on guides (73C40192) and insert into spring housings.

8. Insert two pins (77B40004), in charger block.

9. Crank slide assembly back until slide safety (77C40002) can be installed. Install it.

10. Engage rear slide guide to slide and repin.

11. Locate the rail slide guide (73B40213) and fasten in position.

12. Disengage slide safety and pin in place.

13. Remove slide support.

14. Remove pin in buffer lug.

15. Remove sear holder.

16. Remove two pins in charger block.

17. Felieve load on slide safety in 8. above and remove it. CAUTION: Crank will have a spring load which must be relieved gradually.

18. Align central guide and gas cylinder, securing them in place with their respective nuts.

F. Firing Pin Actuator Sub Assy (76F40268)

1. Slip solenoid mount (75C40137) over housing (75D40138).

2. Align holes and insert tow studs (76B40241) finger tight.

3. Insert two pins (75A40132) into guide (75B40136).

4. Install striker (75B40135, spring (75B40134) and guide into housing and cam into captive position.

5. Rotate housing so two studs are up and install screw (MS51962-52) in tapped hole between the two studs.

6. Place shim (76B40245), plunger (75C40133), spring (see 11701134) and solenoid into mount and secure with nut (MS 35649-202) and cap screw (MS16995-36).

7. Rotate housing 180° and install spring (75A40185) on long end of pin (76A40233).

8. Insert pin with spring down into hole in solenoid mount.

9. Using cap screws (MS16995-42), assemble cover (76A40234), pin holder (76A40235), pin (76A40269), and block (76A40236) together in that order.

10. Locate 9. over pin in 8. above and secure to mount with cap screw.

11. Tighten studs (76Br0241) and firing mechanism to backplate.

G. Assembly and Calibration of Feeder

1. Screw on front bracket.

2. Insert rod (73B40168).

3. From front, insert rod (73B40248).

4. Screw on retainer pin (1 of 4) (7340166).

5. Slip on torsion spring, feed arm, feed release.

6. Screw retainer pin (2 9f 4) thru feed arm.

7. Screw retainer pin (3 of 4) thru feed release making sure front rod is captured. Check rotation of feed arm to assure that front rod is not 180° off.

8. Screw on rear bracket.

9. Slip in spring (73B40165).

10. Screw retainer pin (4 of 4) thru rear bracket, compressing spring (73B40165).

11. Screw on cartridge guide (to receiver).

12. Screw on rammer base.

H. Assembly Procedure, Magazine

1. Begin with housing weldment.

2. Insert spring with the end loop in the upper hole.

3. Place the follower assembly in the lower chute, against the stop.

4. Insert the shaft, short end first.

5. Put arm on the shaft and rotate CCW until spring is wound 2-1/2 turns.

6. Insert the hinge pin (MS9848-20) thru the end holes in the arm and the follower assembly. Add the cotter pin (inside).

7. Add the hub assy and screw on, with 9 self-tapping screws.

8. Test by loading 25 dummy rounds. Rounds should feed freely.

OPERATION OF FIXTURE

1. Push in pinion crank to engage rack. Lock in position with latch.
2. Withdraw pip pin from slide safety lock.
3. Crank back slide until slide sear engages receiver.
4. Disengage charger pinion from rack, slide sear holds back slide along with safety lock.
5. Position main mass charger on rear support and crank back until main sear engages the receiver rail. Manually pull back feeder and insert cartridge.
6. Remove main mass charger and repin slide safety lock.
7. Main sear is released by activating the air cylinder.
8. The slide sear is released by being tripped by a properly positioned plate under the slide.

PARTS LIST	U.S. ARMY WEAPONS COMMAND ROCK ISLAND, ILL, 61201	CONTRACT NO.	CODE IDENT 19204	PL 77F40007	ORIGINAL DATE 2 Feb 77

LIST TITLE ASSEMBLY RECOIL ACT 30	AUTHENTICATION	REV AUTH NO.	SHEET 1 OF 8 SHEETS

ITEM NO.	QTY REQD	CODE IDENT	DRAWING OR DOCUMENT NUMBER	PART OR IDENTIFYING NUMBER	NOMENCLATURE OR DESCRIPTION	PL	REMARKS
			76F40268		Firing Mechanism	+	
			76F40244		Magazine Assembly 5 RD	+	
			73F40135		Receiver	-	
			75D40088		Charging Assembly	+	
	2		73C40215		Housing Guide	-	
			75C40187		Charger Block		
			73C40208		Central Guide	-	
			73F40149		Slide	-	
			73D40147		Piston	-	
			73D40210		Gas Cylinder	-	
			73C40217		Forward Guide	-	
			73D40225		Front Support	-	
			73F40137		Barrel	-	
	2		76B40231		Strap	-	
			76B40229		Spacer Front Support	-	
			142240		Range Table	-	

LTR	DATE	REVISION	LTR	DATE	REVISION	LTR	DATE	REVISION

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ITEM NO.	QTY REQD	CODE IDENT	DRAWING OR DOCUMENT NUMBER	PART OR IDENTIFYING NUMBER	NOMENCLATURE OR DESCRIPTION	PL	REMARKS
			75C40182		Rear Brace		
					Air Cylinder (FABCO-AIR CO. Model "AA-721-0 (Pull only)"		
			76D40270		Firing Pin Assy	+	
	2		73B40136		Rail		
			73C40144		Slide Guide		
			73B40238		Spring, Kickout		
			73B40235		Shaft		
			73B40299		Handle Assy		
			73A40244		Bolt Crank		
			73B40245		Handle		
			73B40246		Crank		
			73C40297		Backplate Assy	+	
			MS9164-166		Pin		
			73B40177		Screw, Alignment		
			MS51021-56		Set Screw		
			73B40159		Stud		

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ITEM NO.	QTY REQD	CODE IDENT	DRAWING OR DOCUMENT NUMBER	PART OR IDENTIFYING NUMBER	NOMENCLATURE OR DESCRIPTION	PL	REMARKS
			73B40158A		Roller, Inner		
			73B40158B		Roller, Outer		
			73B40161		Pawl, Stop		
			73B40160		Pivot		
			73C40162		Spring		
			MS51045-49		Setscrew		
			73B40138		Nut, Headspace		
			73D40216		Mount, Spring		
			73C40215		Housing, Spring, Guide		
			73B40209		Nut, Guide		
			73B40211		Nut, Cylinder		
			73B40207		Seal, Gas		
			73B40212		Plug, Gas Cylinder		
			73C40154		Slide Weldment		
			73B40153		Upright, Slide		
	2		MS24678-41		Screw		
			73C40243		Rack		

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ITEM NO.	QTY REQD	CODE IDENT	DRAWING OR DOCUMENT NUMBER	PART OR IDENTIFYING NUMBER	NOMENCLATURE OR DESCRIPTION	PL	REMARKS
	6		73A40236		Screw, Rack		
			73B40298		Sear, Slide Assy		
			73C40151		Sear		
			73B40191		Spring		
			73A40189		Pin, Trail		
			73A40190		Pin, Spring		
			73B40148		Pin, Pivot		
			73C40150		Spring, Torsion		
			MS17985-623		Pin, Pip		
			73B40213		Slide Guide		
	2		MS16998-96		Screws		
			MS51838-218		Pin, Retaining		
	2		73B40202		Spring, Inner		
	2		73B40201		Spring, Outer		
	2		73C40192		Guide, Spring		
	2		73C40169		Bracket		
			73B40168		Rod, Pivot		

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ITEM NO.	QTY REQD	CODE IDENT	DRAWING OR DOCUMENT NUMBER	PART OR IDENTIFYING NUMBER	NOMENCLATURE OR DESCRIPTION	PL	REMARKS
			73B40248		Rod, Feed Release		
	4		73B40166		Pin, Rod		
			73C40247		Spring, Feeder (M)		
			73C40220		Feed Arm Assy		
			73D40218		Arm, Feed		
			73C40219		Holder, Cartridge		
			73B40167		Release, Feed		
			73B40165		Spring		
	8		MS16998-71		Screw		
			73A40163		Screw, Cartridge Guide		
			73C40164		Guide, Cartridge		
			73B40194		Base, Rammer		
			MS35338-45		Washer		

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ROCK ISLAND ARSENAL
ROCK ISLAND, ILL, 61201

CONTRACT NO.

CODE IDENT 19204

PL 77F40007

ORIGINAL DATE 2 Feb 77

LIST TITLE: ASSEMBLY RECOIL ACT 30

AUTHENTICATION

REV AUTH NO.

SHEET 6 OF 8 SHEETS

ITEM NO.	QTY REQD	CODE IDENT	DRAWING OR DOCUMENT NUMBER	PART OR IDENTIFYING NUMBER	NOMENCLATURE OR DESCRIPTION	PL	REMARKS
			MS16998-57		Screw		
			73B40195		Bolt, Rammer		
	2		MS16998-72		Screw		
	2		MS35338-46		Washer		
			73B40196		Spring		
			73B40197		Rammer		
	3		MS51968-11		Nut		
			73B40198		Stop, Rammer		
			73B40221		Cap, Support		
			74F40215		Test Fixture Assy		
			74F40142		Receiver, etc. Group (4 sh)		
			75B40118		Spring, Leaf		
			75B40119		Block Pivot		
			75B40160		Stud Shoulder		
			75B40161		Spring Torsion		
			75B40188		Latch Charger		
			75B40189		Block		MOD 73C40241

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ITEM NO	QTY REQD	CODE IDENT	DRAWING OR DOCUMENT NUMBER	PART OR IDENTIFYING NUMBER	NOMENCLATURE OR DESCRIPTION	PL	REMARKS
			76B40230		Pin		
			76B40232		Stud		
			76B40242		Stand Off		
			77B40001		Strap Receiver		
			77B40004		Pin Guide		
			77B40006		Plug		
			75C40163		Slide (MOD)		MOD 73F40149
			76C40243		Receiver (MOD)		MOD 73F40135
			73B40142		Lug		
			73C40259		Tool, Combination		
			73C40242		Pinion		
			73C40224		Rear Support		
			75D40122		Rear Support (RT)		
			73C40170		Spring, Feed		
			75C40120		Sear		
			75C40121		Plate, Retainer		
			75C40162		Lock, Slide		

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ITEM NO	QTY REQD	CODE IDENT	DRAWING OR DOCUMENT NUMBER	PART OR IDENTIFYING NUMBER	NOMENCLATURE OR DESCRIPTION	PL	REMARKS
			77C40005		Support, Slide		
			77C40002		Safety, Slide		
			75F40164		Receiver (MOD)		MOD 73F40135
			77C40003		Sear Holder		
			MS24585	24585-1118	Spring		

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CONTRACT NO.	CODE IDENT	PL	ORIGINAL DATE
	19204	76F40268	1 Feb 77

LIST TITLE	AUTHENTICATION	REV AUTH NO.	SHEET 1 OF 2 SHEETS
Firing Mechanism Sub-Assembly			

ITEM NO	QTY REQD	CODE IDENT	DRAWING OR DOCUMENT NUMBER	PART OR IDENTIFYING NUMBER	NOMENCLATURE OR DESCRIPTION	PL	REMARKS
			75D40138		Housing		
	4		MS16995	MS16995-42	Cap Screw		
			76A40234		Cover		
			76A40233		Pin		
			75A40185		Spring		
			76A40235		Pin Holder		
			76A40269		Pin		
			76A40236		Block		
			75B40135		Striker		
			75B40134		Spring		
			75B40136		Spring Guide		
	2		75A40132		Pin		
			76B40245		Shim Solenoid		
			MS35649	MS35649-202	Nut		
			MS51962	MS51962-52	Set Screw		
	8		MS35338	MS35338-43	Lock Washer		
			75C40133		Plunger		

LTR	DATE	REVISION	LTR	DATE	REVISION	LTR	DATE	REVISION

PARTS LIST	DEPARTMENT OF THE ARMY ROCK ISLAND ARSENAL ROCK ISLAND, ILL, 61201	CONTRACT NO.	CODE IDENT 19204	PL 76F40268	ORIGINAL DATE 1 Feb 77
LIST TITLE Firing Mechanism Sub-Assembly		AUTHENTICATION		REV AUTH NO.	SHEET 2 OF 2 SHEETS

ITEM NO.	QTY REQD	CODE IDENT	DRAWING OR DOCUMENT NUMBER	PART OR IDENTIFYING NUMBER	NOMENCLATURE OR DESCRIPTION	PL	REMARKS
			75C40137		Solenoid Mount		
			11701134	11701134	Solenoid		
			MS16995	MS16995-36	Cap Screw		
	2		76B40241		Stud		

LTR	DATE	REVISION	LTR	DATE	REVISION	LTR	DATE	REVISION

SARPI Form 736-3, 1 Nov 73

PARTS LIST

DEPARTMENT OF THE ARMY
ROCK ISLAND ARSENAL
ROCK ISLAND, ILL, 61201

CONTRACT NO.	CODE IDENT	PL	ORIGINAL DATE
	19204	76F40244	1 Feb 77

LIST TITLE	AUTHENTICATION	REV AUTH NO.	SHEET 1 OF 1 SHEETS
Magazine Sub-Assembly 5 Rd			

ITEM NO	QTY REQD	CODE IDENT	DRAWING OR DOCUMENT NUMBER	PART OR IDENTIFYING NUMBER	NOMENCLATURE OR DESCRIPTION	PL	REMARKS
			76C40222		Spring		
			76C40223		Cover		
			76F40226		Magazine Weldment	+	
			MS17984	MS17984-324	Pin Quick Release		
			76C40220		Round Follower		
	4		76B40219		Plunger		
	2		76B40218		Spring Helical		

LTR	DATE	REVISION	LTR	DATE	REVISION	LTR	DATE	REVISION

SARFI Form 736-3, 1 Nov 73

PARTS LIST	DEPARTMENT OF THE ARMY ROCK ISLAND ARSENAL ROCK ISLAND, ILL, 61201	CONTRACT NO.	CODE IDENT 19204	PL 76F40226	ORIGINAL DATE 1 Feb 77
LIST TITLE MAGAZINE WELDMENT		AUTHENTICATION		REV AUTH NO.	SHEET 1 OF 1 SHEETS

ITEM NO	QTY REQD	CODE IDENT	DRAWING OR DOCUMENT NUMBER	PART OR IDENTIFYING NUMBER	NOMENCLATURE OR DESCRIPTION	PL	REMARKS
1	1		76D40224		Support		
2	1		76C40221		Backplate		
3	1		76F40225		Magazine		

LTR	DATE	REVISION	LTR	DATE	REVISION	LTR	DATE	REVISION

SARPI Form 736-3, 1 Nov 73

PARTS LIST	DEPARTMENT OF THE ARMY ROCK ISLAND ARSENAL ROCK ISLAND, ILL, 61201	CONTRACT NO.	CODE IDENT 19204	PL 75D40088	ORIGINAL DATE 1 Feb 77
LIST TITLE Charging Assembly		AUTHENTICATION		REV AUTH NO.	SHEET 1 OF 1 SHEETS

ITEM NO.	QTY REQD	CODE IDENT	DRAWING OR DOCUMENT NUMBER	PART OR IDENTIFYING NUMBER	NOMENCLATURE OR DESCRIPTION	PL	REMARKS
			75C40083		Side Plate (lt)		
			75C40084		Side Plate (rt)		
	16		MS16995	MS16995-52	Cap Screw		
			75B40080		Drive Nut		
			75C40082		End Plate		
			75B40081		Drive Stud		
	2		75B40086		Tension Plate		
			MS17984	MS17984-C430	Pin, Quick Release		
			75B40085		Drive Plate		
			75C40087		Charger Plate		

LTR	DATE	REVISION	LTR	DATE	REVISION	LTR	DATE	REVISION

SARPI Form 736-3, 1 Nov 73

PARTS LIST	U.S. ARMY WEAPONS COMMAND ROCK ISLAND, ILL, 61201	CONTRACT NO.	CODE IDENT 19204	PL 76D40270	ORIGINAL DATE 4 Feb 77
LIST TITLE FIRING PIN ASSEMBLY		AUTHENTICATION		REV AUTH NO.	SHEET 1 OF 1 SHEETS

ITEM NO.	QTY REQD	CODE IDENT	DRAWING OR DOCUMENT NUMBER	PART OR IDENTIFYING NUMBER	NOMENCLATURE OR DESCRIPTION	PL	REMARKS
1	1		73B40183		Cross Pin		
2	1		76C40237		Bolt Extension		MOD 73C40187
3	2		73A40176		Pin		
4	1		73B40186		Base		
5	1		73B40181	73B40181-C	Spring		
6	1		76B40239		Tube		
7	1		76B40238		Bolt		MOD 73D40132
8	1		75A40183		Spring		
9	1		76A40240		Firing Pin		MOD 73B40185
10	1		73D40139		Chamber		
11	1		73C40297		Back Plate Assy	X	

LTR	DATE	REVISION	LTR	DATE	REVISION	LTR	DATE	REVISION

AMSWE Form 688, 21 AUG 70 (TEMP)

PARTS LIST	U.S. ARMY WEAPONS COMMAND ROCK ISLAND, ILL, 61201	CONTRACT NO.	CODE IDENT 19204	PL 73C40297	ORIGINAL DATE 4 Feb 77

LIST TITLE	AUTHENTICATION	REV AUTH NO.	SHEET 1 OF 1 SHEETS
BACKPLATE ASSY			

ITEM NO.	QTY REQD	CODE IDENT	DRAWING OR DOCUMENT NUMBER	PART OR IDENTIFYING NUMBER	NOMENCLATURE OR DESCRIPTION	PL	REMARKS
1	1		73F40146		Backplate		
2	4		73B40172		Plunger		
3	4		73C40227	73C40227-B	Spring		
4	4		73B40141		Plug		

LTR	DATE	REVISION	LTR	DATE	REVISION	LTR	DATE	REVISION

AMSWE Form 688, 21 AUG 70 (TEMP)

INPUT PARAMETERS

1.	PRMT(1)	Initial integration time (sec)
2.	PRMT(2)	Final integration time (sec)
3.	PRMT(3)	Integration time step (sec)
4.	PRMT(4)	Allowed error (dimensionless)
5.	PRMT(5)	Control number (set to zero)
6.	QRANG(1)	Expected displacement range of receiver (in)
7.	QRANG(2)	Expected displacement range of slide (in)
8.	QDRANG(1)	Expected velocity range of receiver (in/sec)
9.	QDRANG(2)	Expected velocity range of slide (in/sec)
10.	Q(1)	Initial displacement of receiver (in)
11.	Q(2)	Initial displacement of slide (in) (if not in locked position)
12.	QD(1)	Initial velocity of receiver (in/sec)
13.	QD(2)	Initial velocity of slide (in/sec)
14.	TRANG	Expected temperature range of cylinder gas (°R)
15.	DELC, δ_C	Angle chamber is when it becomes locked (rad)
16.	QCFL, q_{CFL}	Displacement of chamber when fully locked (in)
17.	QCIL, q_{CIL}	Displacement of chamber when locking starts (in)
18.	EPSCI, ϵ_{CI}	Distance to represent sharp change in slope (in)

19.	BCI, b_{CI}	Minor axis of ellipse for "sharp slope" (in)
20.	BCF, b_{CF}	Minor axis of ellipse for "sharp slope" (in)
21.	UFL, U_{FL}	(q_2-q_c) when chamber is fully locked (in)
22.	UIL, U_{IL}	(q_2-q_c) when chamber is starting to lock (in)
23.	EPSQF, ε_{qf}	Distance to describe a sharp change in slope (in)
24.	BQF, b_{qf}	Minor axis of ellipse for "sharp slope" (in)
25.	ERROR, ε_c	Error allowed for computing constraint function (rad)
26.	DELA, δ_a	Distance slide travels before gas is cut off (in)
27.	DELB, δ_b	Distance slide travels when gas flows back to barrel (in)
28.	AO, A_o	Throat area (in^2)
29.	CO, C_o	Orifice coefficient (dimensionless)
30.	CVB, C_{vb}	Specific heat at constant volume of barrel gas $\frac{in^2}{Sec^2\text{-}°R}$
31.	CPB, C_{pb}	Specific heat at constant pressure of barrel gas $\frac{in^2}{Sec^2\ °R}$
32.	VO, V_o	Initial cylinder volume (in^3)
33.	ACYL, A_c	Cylinder cross sectional area (in^2)

34. PA, P_a Atmospheric pressure (PSI)

35. TA, T_a Atmospheric temperature (°R)

36. TPORT, T_{port} Time for projectile to reach port (sec)

37. TBM, T_{bm} Temperature of chamber gas when projectile leaves barrel (°R)

38. PBM, P_{bm} Pressure of chamber gas when projectile leaves barrel (PSI)

39. NP # of discrete (time vs chamber pressure) points

40. TIMEP(I) Time coordinates (sec) after projectile passes port

41. PB(I) Pressure coordinates (PSI) after projectile passes port

42. NT # of (time vs chamber temperature) coordinates

43. TIMET(I) . Time coordinates (sec) after projectile passes port

44. TB(I) Temperature coordinates (°R) after projectile passes port

45. NCHAM # of coordinates describing chamber pressure force

46. CHAMT(I) Time coordinates (sec)

47. CHAMF(I) Force coordinates (lb)

48. Q1REF, q_{1ref} q_1 - coordinates describing recoil force (in)

49. F1REF, F_{1ref} Force corresponding to q_{iref} (lb)

50. Q2RFF, q_{2ref} q_1 - coordinates describing recoil force (in)

51. F2REF, F_{2ref} Force corresponding to q_{2ref} (lb)

52.	Q3REF, q_{3ref}	q_1 - coordinates describing recoil force (in)
53.	F3REF, F_{3ref}	Force corresponding to q_{3ref} (lb)
54.	CR2, C_{r2}	Viscous frictional coefficient(receiver) $(\frac{lb\text{-}sec}{in})$
55.	PLDF, $F_{pld(f)}$	Preload force on forward buffer (receiver) (lb)
56.	QOF, $q_{o(f)}$	q_1 - coordinates when receiver contacts forward buffer (in)
57.	PELSF, % loss (f)	Percent energy loss from impact of forward buffer (%)
58.	XK1F, $K_{1(f)}$	Spring rate of forward buffer $(\frac{lb}{in})$
59.	EPSF, $E_{(f)}$	Distance required for force to build up to the preload (in)
60.	PLDR, $F_{pld(r)}$	Preload force on rear buffer (receiver) (lb)
61.	QOR, $q_{o(r)}$	$(-q_1)$ - coordinates when receiver contacts rear buffer (in)
62.	PELSR, % loss (r)	Percent energy loss from impact of rear buffer (%)
63.	XK1R, $K_{1(r)}$	Spring rate of receiver rear buffer $(\frac{lb}{in})$
64.	EPSR, $E_{(r)}$	Distance required for force to build up to preload of rear buffer (in)
65.	TDELAY, T_{delay}	Delay between firing and chamber force starting (sec)
66.	FSREF, F_{sref}	Slide drive spring preload (lb)
67.	XKS1, K_{s1}	Spring rate of slide drive spring $(\frac{lb}{in})$

68.	CS2, C_{s2}	Viscous frictional coefficient (slide) $(\frac{lb\text{-}sec}{in})$
69.	PLDSR, $F_{pld(sr)}$	Preload force on slide rear buffer (lb)
70.	QOSR, $Q_{o(sr)}$	$(-q_2)$ - coordinate when slide contacts rear buffer (in)
71.	PELSSR, % loss (sr)	Percent energy loss from impact of rear buffer spring
72.	XK1SR, $K_{1(sr)}$	Spring rate of slide rear buffer spring $(\frac{lb}{in})$
73.	EPSSR, $E_{(sr)}$	Distance required for force to build up to preload $(\frac{lb}{in})$
74.	PLDSS, $F_{pld(ss)}$	Preload force on sear buffer spring (lb)
75.	QOSS, $Q_{o(ss)}$	q_2 - coordinate when slide contacts sear buffer (in)
76.	PELSSS, % $loss_{(ss)}$	Percent energy loss from impact of sear buffer
77.	XK1SS, $K_{1(ss)}$	Spring rate of sear buffer
78.	EPSSS, $E_{(ss)}$	Distance required for force to build up to preload (in)
79.	PLDSF, $F_{pld(sf)}$	Preload on slide forward buffer spring (lb)
80.	QOSF, $Q_{o(sf)}$	q_2 - coordinate when slide contacts forward buffer (in)
81.	PELSSF, % loss (sf)	Percent energy loss from impact of forward buffer
82.	XK1SF, $K_{1(sf)}$	Spring rate of forward buffer $(\frac{lb}{in})$

83.	EPSSF, $E_{(sf)}$	Distance required for force to build up to preload (in)
84.	PLDCR, $F_{pld(cr)}$	Preload on chamber rear buffer
85.	QOCR, $Q_{o(cr)}$	$(-q_c)$ - coordinate when chamber contacts rear buffer (in)
86.	PELSCR, % $loss_{(cr)}$	Percent energy loss from impact of rear buffer spring
87.	XK1CR, $K_{1(cr)}$	Spring rate of chamber rear buffer spring $(\frac{lb}{in})$
88.	EPSCR, $E_{(cr)}$	Distance required for force to build up to the preload (in)
89.	CC, C_c	Viscous frictional coefficient for chamber $(\frac{lb\text{-}sec}{in})$
90.	G	Gravitational constant $(\frac{in}{sec^2})$
91.	THETA	Initial elevation angle (not applicable in model with vehicle motion)(Rad)
92.	Q1REL	Position of receiver when slide is released
93.	T1	Time delay (start of chamber force to peak value)(sec)
94.	NINT	Printing frequency of forces, pressures, etc.
95.	IDELAY	Control # 1 - if correcting for time delay in firing logic 0 - if approximating recoil force with non-correctant function
96.	IPCNT	Control # (1 if approximating recoil force with a constant function)

97.	IECNT	Control # (1 if correcting for elevation)
98.	IICNT	Control # (1 if correcting for inertial forces)
99.	IWRITE	Control # (1 if desire output on disk)
100.	ILOCK	Control # (1 if chamber starts off being locked)
101.	WSL	Weight of slide (lb)
102.	WCH	Weight of chamber (lb)
103.	WREC	Weight of receiver (lb)
104.	CIXX	Moment of inertia of chamber (about translational axis)(lb-in-sec^2)
105.	Q2PTI(I)	q_2 - coordinate when "$\dot{z}$th" constant dissipative force begins (in)
106.	Q2PTI1(I)	q_2 - coordinate when "$\dot{z}$th" constant dissipative force ends (in)
107.	EPSQ2I(I)	Build up distance for "$\dot{z}$th" force (slide)(in)
108.	FRICQ2(I)	Value of force (slide)(lb)
109.	ICNQ2I(I)	Control # (if 1, equal and opposite force acts on receiver)
110.	NQ2	Number of forces opposing when velocity is negative
111.	MQ2	Number of forces opposing when velocity is positive
112.	Q1PTI(I)	q_1 - coordinate when "$\dot{z}$th" constant dissipative force begins (in)
113.	Q1PTI1(I)	q_1 - coordinate when "$\dot{z}$th" constant dissipative force ends (in)

114.	EPSQ1I(I)	Build up distance for "żth" force (receiver)(in)
115.	FRICQ1(I)	Value of force (receiver)(lb)
116.	ICNQ1I(I)	Control # (no equal and opposite force acting) Arbitrary
117.	NQ1	Number of forces opposing motion when velocity is negative
118.	MQ1	Number of forces opposing motion when velocity is positive
119.	QCPTI(I)	q_c - coordinate when "żth" constant dissipative force begins (in)
120.	QCPTI1(I)	q_c - coordinate when "żth" constant dissipative force ends (in)
121.	EPSQCI(I)	Build up distance for "żth" force (in)
122.	FRICQC(I)	Value of force (chamber) (lb)
123.	ICNQCI(I)	Control # (1 if equal and opposite force acts on receiver)
124.	NQC	Number of forces opposing motion when velocity is negative
125.	MQC	Number of forces opposing motion when velocity is positive
126.	TCPTI(I)	θ_c - coordinate when "żth" constant dissipative force begins (rad)
127.	TCPTI1(I)	θ_c - coordinate when "żth" constant dissipative force ends (rad)
128.	EPSTCI(I)	Build up distance for "żth" force (rad)

129.	FRICTC(I)	Value of torque (in-lb)
130.	ICNTCI(I)	Control # (no equal and opposite force) Arbitrary
131.	NTC	Number of forces opposing negative angular velocity
132.	MTC	Number of forces opposing positive angular velocity
133.	UFR	Distance slide is forward before weapon can fire
134.	ISTATE	Printing frequence of state variables
135.	IFILE	File # for data to be plotted
136.	IFILED	File # for CRT if running from that terminal (use 6 for printer and 10 for CRT)
137.	IMEAS	Set equal to 1 if the C.M. is used in the control equations (otherwise receiver state is the measured variable)
138.	ISPECI	If using center of mass as the measured variable for control equation and we desire the state of the center of mass to not be returned to the initial displacement with zero velocity, then set to one
139.	QCMS	If ISPECI = 1, the center of mass will return to this displacement after firing
140.	QDCMS	If ISPECI = 1, the center of mass will return to this velocity after firing
141.	NOTAR	Set equal to one if no specified target
142.	XTAR	x - coordinate of the target (m)

143.	YTAR	y - coordinate of the target (m)
144.	ZTAR	z - coordinate of the target (m) NOTE: See "FUE Air Defense Evaluation" for flight path parameters
145.	THETAO	Initial elevation angle (if no target) (rad)
146.	THETPO	Initial elevation rate (if no target) (rad/sec)
147.	THEPPP	Elevation acceleration (if no target) (rad/sec^2)
148.	PSIO	Initial azimuth angle (if no target) (rad)
149.	PSIPO	Initial azimuth rate (if no target) (rad/sec)
150.	PSIPPP	Azimuth acceleration (if no target) (rad/sec^2)
151.	XSP	x - coordinate of slide C.G. relative to receiver C.G. (initial)(in)
152.	YSP	y - coordinate of slide C.G. relative to receiver C.G. (initial)(in)
153.	ZSP	z - coordinate of slide C.G. relative to receiver C.G. (initial)(in)
154.	XRP	x - coordinate of receiver C.G. relative to center of rotation of the turret (initial)(in)
155.	YRP	y - coordinate of receiver C.G. relative to center of rotation of the turret (initial)(in)
156.	ZRP	z - coordinate of receiver C.G. relative to center of rotation of the turret (initial)(in)

157.	XCP	x - coordinate of chamber C.G. relative to receiver C.G. (initial)(in)
158.	YCP	y - coordinate of chamber C.G. relative to receiver C.G. (initial)(in)
159.	ZCP	z - coordinate of chamber C.G. relative to receiver C.G. (initial)(in)
160.	CIYY	Principle moment of inertia of the chamber (lb-in-sec^2)
161.	CIZZ	Principle moment of inertia of the chamber (lb-in-sec^2)

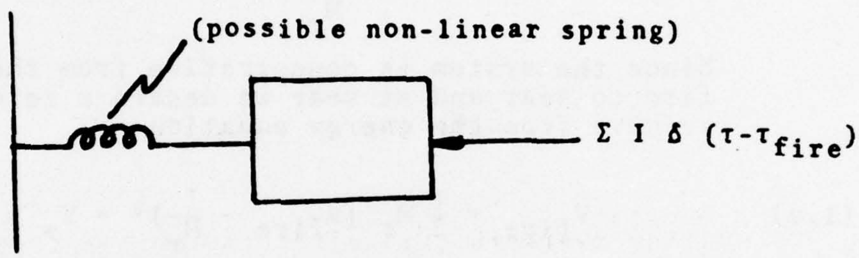
FLIGHT PATH DATA

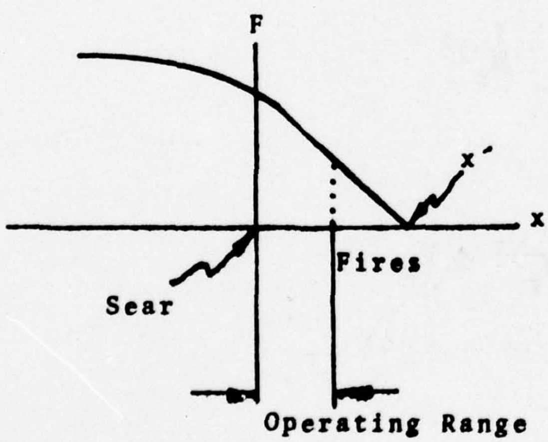
1.	NSEG	Number of segments in flight path
2.	X(1)	x coordinate at start of flight (m)
3.	Y(1)	y coordinate at start of flight (m)
4.	Z(1)	z coordinate at start of flight (m)
5.	THETA(1)	Heading coordinate at start of flight (degrees)
6.	VS(1)	Horizontal velocity
7.	VZ(1)	Vertical velocity
8.	TSTART(1)	Time at start of flight path (sec)
9.	ITYPE	Control # Line segment = 1 Circular segment = 2 Spiral segment = 3
10.	AZ	Vertical acceleration (g)
11.	S	Segment length (m) (if ITYPE = 1) Turn angle (degrees) (if ITYPE ≠ 1)
12.	AS	Horizontal acceleration (g) (if ITYPE = 1) Radial accelration (g) (if ITYPE ≠ 1)
13.	RAD	Turn flag for ITYPE = 2 or 3 1 = right -1 = left
14.	VDOT	Tangential accelration (g) (ITYPE = 3)

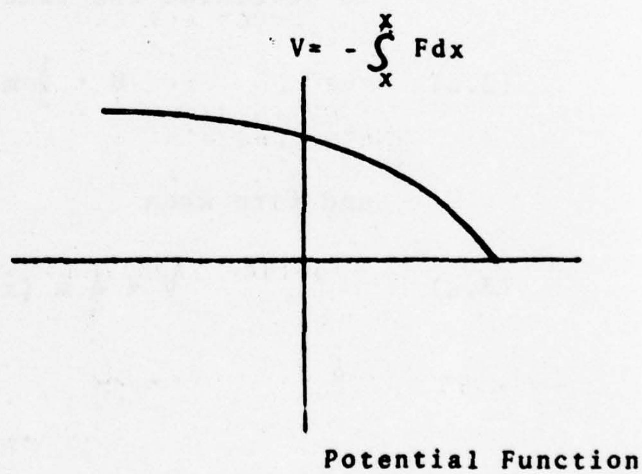
CONTROL ANALYSIS

CONTROL EQUATION ANALYSIS

The concept of measuring the displacement and velocity of the recoiling parts and computing a parameter such that when it becomes greater than zero the gun will fire and return the recoiling parts back to the origin will be developed for a simple system and then corrections will be made to the simple system to account for elevation angle, firing delay, accelerating reference frame, etc.







Typical Force vs. Deflection Curve

Operating range chosen to be as constant as possible.

V_s - Potential at sear

V_{fire} - Potential when fired

X_{fire}, $\dot{X}_{fire}$ - Displacement and velocity at time of fire

Since the impulse applied to the weapon is a delta function, the potential will not have changed at firing and only the velocity will have decreased by

$$- \frac{I}{M_r}$$

Since the system is conservative from the time after fire to sear and at sear we desire a zero velocity, we have from the energy equation:

(1.a) $$V_{fire} + \frac{1}{2} M_r \left(\dot{X}_{fire} - \frac{I}{M_r}\right)^2 = V_s$$

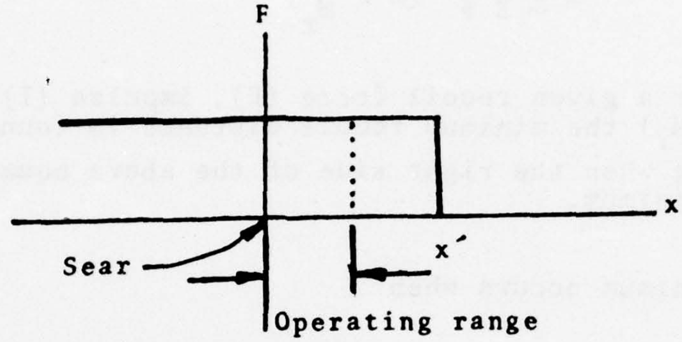
To determine the time of fire, we compute

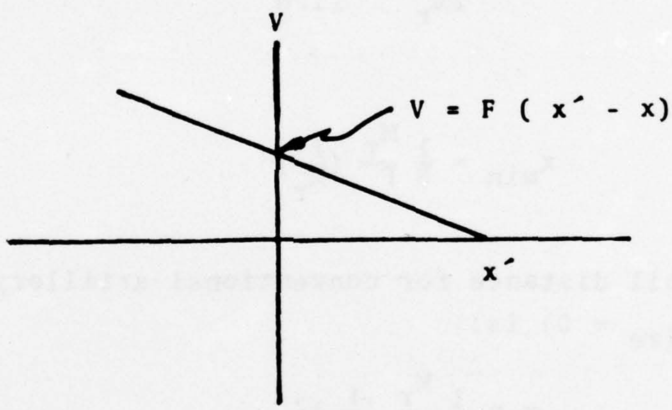
(2.a) $$V + \frac{1}{2} m \left(\dot{x} - \frac{I}{M_r}\right)^2$$

and fire when

(3.a) $$V + \frac{1}{2} m \left(\dot{x} - \frac{I}{M_r}\right)^2 \leq V_s$$

In the case of a constant force:





(4.a) $$V = F\,(x' - x) \quad \text{(in operating range)}$$

From equations (3.a) and (4.a) fire when

(5.a) $$-Fx + \frac{1}{2} M_r \left(\dot{x} - \frac{I}{M_r}\right)^2 \leq 0$$

From equation (5.a) we have:

$$x \geq \frac{1}{2} \frac{M_r}{F} (\dot{x} - \frac{I}{M_r})^2$$

For a given recoil force (F), impulse (I), and mass (M_r) the minimum recoil distance is found by finding when the right side of the above equation is a minimum.

Minimum occurs when

$$x = \frac{I}{2M_r} = \dot{x}_{fire}$$

and

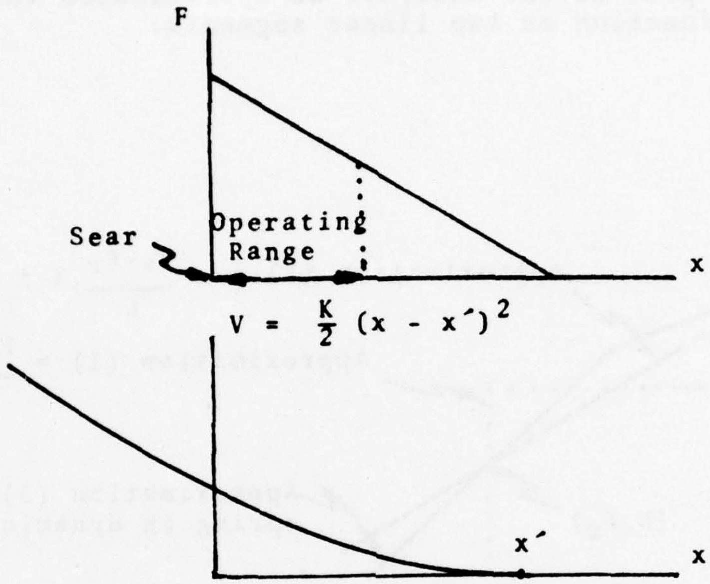
$$x_{min} = \frac{1}{8} \frac{M_r}{F} (\frac{I}{M_r})^2$$

Recoil distance for conventional artillery ($\dot{x} = \dot{x}_{fire} = 0$) is:

$$x = \frac{1}{2} \frac{M_r}{F} (\frac{I}{M_r})^2$$

The minimum is the "firing out of battery principle" and for a given impulse, mass, and recoil force, only requires a 1/4 of the conventional weapon recoil distance.

If the spring has a non-zero slope:



From equation (3.a), fire when

$$(6.a) \qquad -\frac{K}{2}(x')^2 + \frac{K}{2}(x - x')^2 + \frac{1}{2}M_r\left(\dot{x} - \frac{I}{M_r}\right)^2 \le 0$$

or

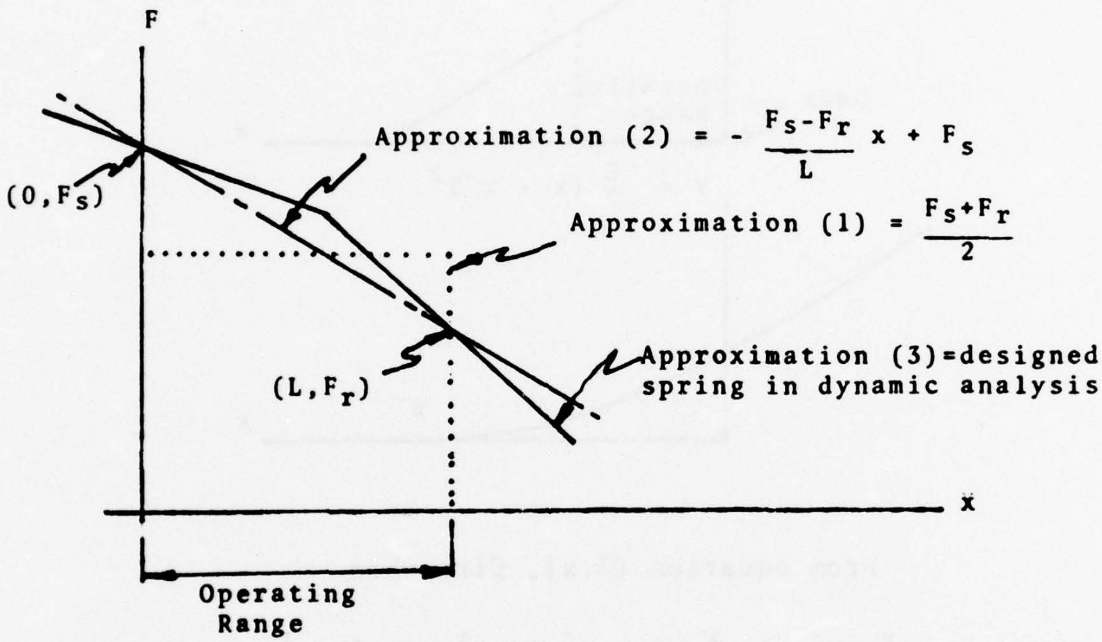
$$(7.a) \qquad \frac{Kx}{2}(x - 2x') + \frac{1}{2}M_r\left(\dot{x} - \frac{I}{M_r}\right)^2 \le 0$$

If $x' \to \infty$ and $Kx' \to F$, then $K \to 0$ and the control equation (7.a) reduces to:

$$(8.a) \qquad -Fx + \frac{1}{2}M_r\left(\dot{x} - \frac{I}{M_r}\right)^2 \le 0$$

(Note: Equation 8.a is the same as 5.a)

The complexity of the control equation increases as we deviate from a constant forcing function. In the main part of the analysis we approximated the forcing function as two linear segments.



To calculate the potential as a function displacement for "approximation (3)" would introduce too much complexity into the control circuitry. The only candidate control functions will be approximations (1) and (2). (i.e., equations (5.a) and (7.a))

In terms of the above parameters, equations (5.a) and (7.a) become,

$$\frac{-(F_s + F_r)}{2} x + \frac{1}{2} M_r (\dot{x} - \frac{I}{M_r})^2 \leq 0 \tag{9.a}$$

$$\frac{F_s - F_r}{L} \frac{x^2}{2} - F_s x + \frac{1}{2} M_r (\dot{x} - \frac{I}{M_r})^2 \leq 0 \tag{10.a}$$

We now consider the corrections required if other forces act on the recoiling mass other than the recoil force.

T_{fire}, V_{fire} - Kinetic and potential energy when weapon is fired

V_s - Potential energy at sear position (Kinetic energy is zero)

W_{fs} - Work done by forces other than recoil force in going from the firing position to sear

From energy equation:

$$W_{fs} = V_s - V_{fire} - T_{fire} \tag{11.a}$$

We compute $W_{xs} + V + T$ until,

$$W_{xs} + V + T \leq T_s \tag{12.a}$$

To determine the time of fire.

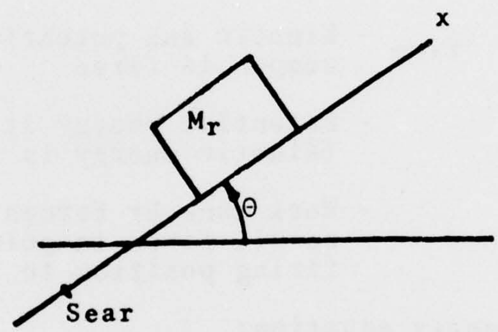
W_{xs} - Work done on mass from forces other than the recoil force in going from x to sear.

If the recoil force is a constant, then the control equation is fire if;

(13.a) $$W_{xs} - F_x + \frac{1}{2} M_r \left(\dot{x} - \frac{I}{M_r}\right)^2 \leq 0$$

The most significant force would be the weight component when the weapon is elevated.

(14.a) $$W_{xs_1} = M_r \, g \, \mathrm{Sin}\, \theta \, x$$

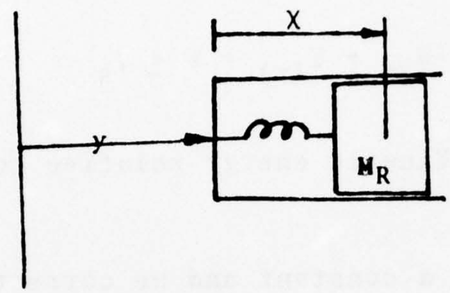


From equations (13.a) and (14.a)

(15.a) $$(M_r \, g \, \mathrm{Sin}\, \theta - F)\, x + \frac{1}{2} M_r \left(\dot{x} - \frac{I}{M_r}\right)^2 \leq 0$$

For a 200 lb weapon and 90° elevation F = 1200 lb. The required correction for the coefficient of x would be $\frac{1}{6}$F which is quite significant. Smaller corrections for friction and stripping of a round might be included.

We now investigate the correction due to an accelerating reference frame.



The control equation is the same as equation (12.a) except we must include in the term W_{xs} the work done by the inertial force, $-M_r\ddot{y}$. To see this we derive the energy equation for a moving reference frame.

(16.a) $$\int M_r\,(\ddot{x} + \ddot{y})\,dx = \int F dx$$

$$F = F_{cons} + F_{ncons}$$

F_{cons} - Conservative force (function of x)

F_{ncons} - Non-conservative force

$\Delta T_{(x)}$ - Change in kinetic energy relative to x-axis

ΔV - Change in potential energy

From equation (16.a) we have

(17.a) $$\Delta T_{(x)} = -\Delta V + \int (F_{ncons} - M_r\,\ddot{y})\,dx$$

The term, $\int (F_{ncons} - M_r \ddot{y})\, dx$, corresponds to what we called W_{xs}, previously. The control equation is, therefore, fire if:

$$W_{xs} + T_{(x)} + V \leq V_s$$

$T_{(x)}$ - Kinetic energy relative to the turret.

If $\ddot{y}$ is a constant and we correct for elevation, and the recoil force is a constant, the control equation is:

fire if,

$$(M_r\ g \operatorname{Sin} \theta - F - M_r \ddot{y})\ x + \frac{1}{2} M_r \left(\dot{x} - \frac{I}{M_r}\right)^2 \leq 0 \tag{18.a}$$

For a 200 lb weapon, and 1-g acceleration, and F = 1200 lb the correction to F would be $\frac{1}{6}F$.

Finally, we consider a time delay that is constant from round to round. Because of the self-correcting nature of the control equation, small variations in time delay will not be detrimental.

T_{delay} - Time between firing and when the round effectively delivers the impulse to the weapon

To determine the firing time we must project ahead the displacement and velocity.

x_m, $\dot{x}_m$ - measured displacement and velocity

F_e - external forces

t_m - time of measurement

(20.a) $$\dot{x} = \dot{x}_m + \int_{t_m}^{t_m + T_{delay}} \frac{F_e}{M_r} \, dt$$

(21.a) $$x = x_m + \dot{x}_m T_{delay} + \int_{t_m}^{t_m + T_{delay}} \frac{F_e}{M_r} (t_m + T_{delay} - \tau) \, d\tau$$

For this correction we assume all forces are constant (recoil, gravitational, and inertial).

(22.a) $$F_e = F - M_r g \operatorname{Sin} \theta - M_r \ddot{y}$$

To correct for ignition delay we add to the measured velocity and displacement at a particular time - a constant.

(23.a) $$\dot{x} = \dot{x}_m + \frac{F_e}{M_r} T_{delay}$$

$$x = x_m + \dot{x}_m T_{delay} + \frac{F_e}{M_r} \frac{(T_{delay})^2}{2}$$

In the computer we allow for making or not making corrections:

I_{delay} = 1 If we correct for time delay

= 0 Otherwise

I_e = 1 If we correct for elevation

= 0 Otherwise

I_I = 1 If we correct for inertial forces

= 0 Otherwise

I_p = 1 If we use a constant function to compute spring potential

= 0 If we use a linear function to compute spring potential

The control numbers allow us to determine the effect on the system if a correction is made or not.

Summary:

(25.a) $$F_e = \frac{F_r + F_s}{2} - I_e M_r g \sin\theta - I_I M_r \ddot{y}$$

$$x = x_m + I_{delay} \left(\dot{x}_m T_{delay} + \frac{F_e}{M_r} \frac{(T_{delay})^2}{2}\right)$$

$$\dot{x} = \dot{x}_m + I_{delay} \left(\frac{F_e}{M_r} T_{delay}\right)$$

Fire if,

$$(25.b) \quad -I_p \frac{(F_s + F_r)}{2} x + (1 - I_p) \left(\frac{F_s - F_r}{L} \frac{x^2}{2} - F_s x\right)$$

$$+ (I_e M_r g \sin\theta + I_I M_r \ddot{y}) x$$

$$+ \frac{1}{2} M_r (\dot{x} - \frac{I}{M_r})^2 \leq 0$$

What has been called x in the previous analysis is the center of mass of the system with respect to the turret. If the moving parts inside the weapon are light, then we can measure the receiver displacement and velocity, otherwise we need to measure the center of mass of the system.

Q_{cm_o} - Displacement we desire the center of mass returned to after firing.

$\dot{Q}_{cm_o}$ - Velocity of the center of mass we desire the center of mass returned to.

$$(26.a) \quad x = Q_{cm} - Q_{cm_o}$$

$$\dot{x} = \dot{Q}_{cm} - \dot{Q}_{cm_o}$$

For example, if we start with the slide and receiver at sear with zero velocity, then

q_{10} - Position of receiver at sear

q_{20} - Position of slide at sear

q_{co} - Position of chamber at sear

(27.a) $Q_{cm_o} = (q_{10}\, W_{rec} + q_{20}\, W_{sl} + q_{co}\, W_{ch}) \,/\, (W_{rec} + W_{sl} + W_{ch})$

$\dot{Q}_{cm_o} = 0$

Since the chamber is always locked to the receiver when the weapon fires, we don't need to measure q_c.

(28.a) $Q_{cm} = [(W_{rec} + W_{sl} + W_{ch})\, q_1 + W_{sl}\, q_2]/(W_{rec} + W_{sl} + W_{ch})$

$\dot{Q}_{cm} = [(W_{rec} + W_{sl} + W_{ch})\, \dot{q}_1 + W_{x1}\, \dot{q}_2]/(W_{rec} + W_{sl} + W_{ch})$

From equations (27.a) and (28.a),

$$\text{(29.a)} \quad x = (q_1 - q_{10}) + \frac{(q_2 + q_{20} - q_{10})\, W_{sl} + (q_{co} - q_{10})\, W_{ch}}{W_{rec} + W_{sl} + W_{ch}}$$

$$\dot{x} = \dot{q}_1 + \frac{W_{sl}\, \dot{q}_2}{W_{rec} + W_{sl} + W_{ch}}$$

From equation (29.a) we see the second term gives the error in measuring only the receiver displacement and velocity as a measure of the center of mass displacement and velocity.

$$W_{rec} + W_{sl} + W_{ch} = 200 \text{ lb}$$

Approximate values:

$$W_{sl} = 32 \text{ lb}$$

$$W_{ch} = 16 \text{ lb}$$

$$q_{10} = 0$$

$$q_{20} = 9 \text{ in}$$

$$q_{co} = 7 \text{ in}$$

When the weapon fires,

$$q_{2} = 0$$

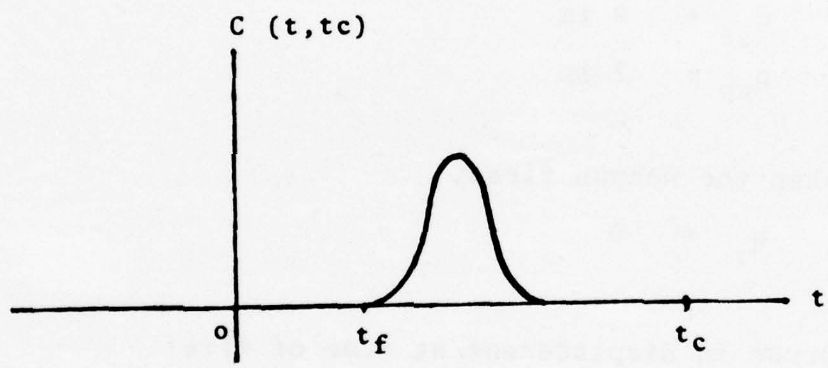
Error in displacement at time of fire:

$$\frac{9(32) + 7(16)}{200} = 2 \text{ in}$$

For five inches of recoil the error would be 40% which is not tolerable. Also, computer results indicate that the impact of the slide into the receiver would give a velocity to the receiver large enough to cause firing of the weapon. Conclusion the receiver cannot be used as a measure of the C.G. of the system.

On page 119 there are two types of time delay; ignition delay (t_{delay}) and a delay (t_1) because of the round impulse being distributed over time rather than being a delta function. We now show how to determine t_1.

$C\ (t,tc)$	-	Chamber force
t_f	-	Firing time
t_c	-	Cycle time
F	-	External forces acting on system other than the chamber force



$$M_r x = F - C\ (t,t_f)$$

For one cycle:

$$(30.a) \quad 0 = M_r \dot{x} \Big|_0^{t_c} = \int_0^{t_c} = \int_0^{t_c} (F - C(t,tf))\ dt$$

$$0 = M_r \, x \Big|_0^{t_c} = \int_0^{t_c} \int_0^{t_c} (F - C(t,tf))\, d\tau \, dt \qquad (31.a)$$

From equations (30.a) and (31.a)

$$\int_0^{t_c} (F - C(t,tf))\, t \, dt = 0 \qquad (32.a)$$

We compute the cycle time assuming $c(t,tf) = I\delta(t - t_f)$ and F = constant.

$$t_c = \frac{I}{F} \qquad (33.a)$$

A nominal firing time is computed based on the same assumptions.

$$t_{f(n)} = \frac{I}{2F} \qquad (34.a)$$

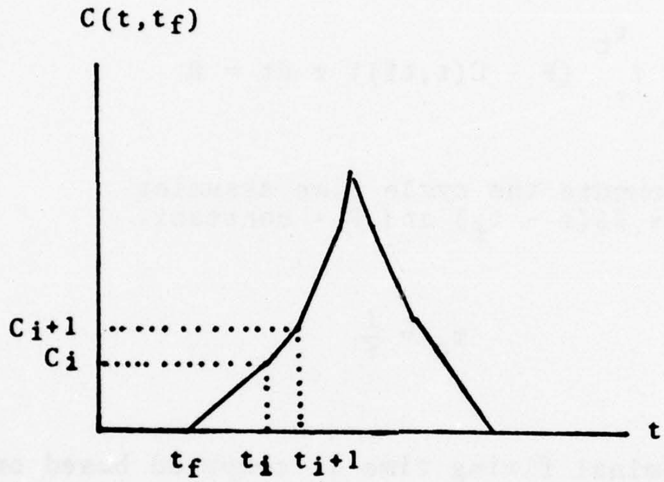
From equation (32.a) and assuming t_c is a constant given by equation (33.a)

$$\frac{F\, t_c^{\,2}}{2} - \int_0^{t_c} C(t,tf)\, tdt = 0 \qquad (35.a)$$

$$t_{f(n)} - t_f = t_1 \qquad (36.a)$$

Equation (36.a) represents the time delay in the system due to the distributed force, $c(t,t_f)$.

We now show how to calculate t_f from equation (35.a).



(37.a) $\quad C(t,tf) = a_i\ (t - t_f) + b_i \qquad t_i \leq t \leq t_i + 1$

From equations (35.a) and (37.a) we have

(38.a) $$\frac{F\ t_c{}^2}{2} = -\ t_f\ \frac{A_i}{2}\ (t_{i+1}{}^2 - t_i{}^2) + \frac{b_i}{2}\ (t_{i+1}{}^2 - t_i{}^2) + \frac{a_i}{3}\ (t_{i+1}{}^3 - t_i{}^3)$$

$$(39.a) \quad t_f = \frac{\frac{b_i}{2}\left(t_{i+1}^2 - t_i^2\right) + \frac{a_i}{3}\left(t_{i+1}^3 - t_i^3\right) - \frac{Ft_c^2}{2}}{\frac{a_i}{2}\left(t_{i+1}^2 - t_i^2\right)}$$

From equations (33.a), (34.a), and (39.a)

$$(40.a) \quad t_1 = \frac{I}{2F} - \frac{\frac{b_i}{2}\left(t_{i+1}^2 - t_i^2\right) + \frac{a_i}{3}\left(t_{i+1}^3 - t_i^3\right) - \frac{I^2}{2F}}{\frac{a_i}{2}\left(t_{i+1}^2 - t_i^2\right)}$$

Note: $a_i = \dfrac{C_{i+1} - C_i}{t_{i+1} - t_i}$

and

$b_i = Y_i - a_i\, t_i$

See special computer program for computing t_1.

t_1 = 10 m sec, computed for chamber force with

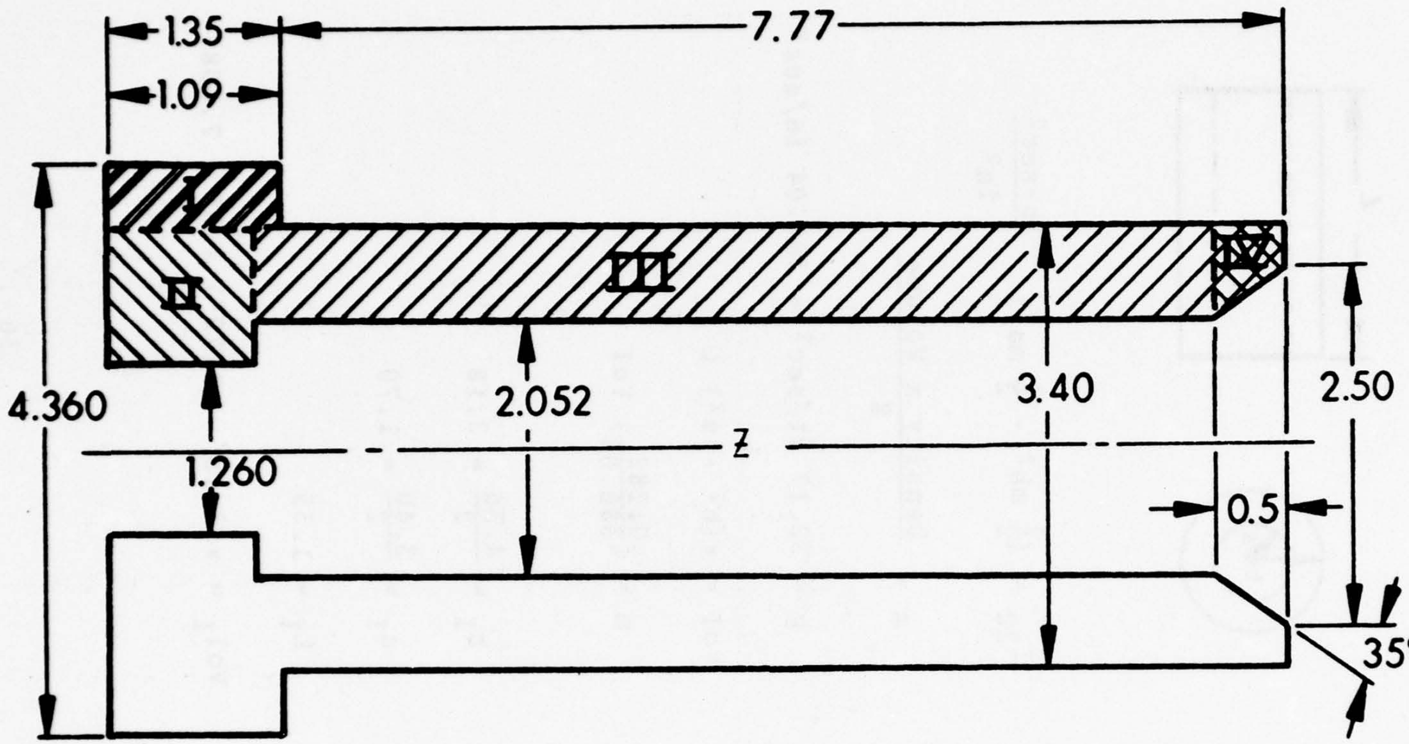
I = 160 lb/sec

F = 1200 lb

and using supplied $C(t,t_f)$.

CHAMBER MOMENT OF INERTIA

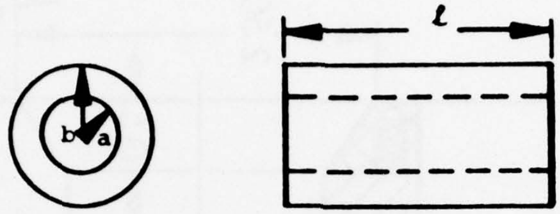
CHAMBER MOMENT OF INERTIA CALCULATION AND MASS



4150 Steel

0.282 lbs/in^3 density

Moment of Inertia Calculation for Chamber.



$$I_{22} = \left(\frac{1}{2} mb^2 - \frac{1}{2} ma^2\right) \quad \frac{\text{lb-sec}^2}{\text{in}^2}$$

$$m = \frac{\text{Density x Volume}}{g}$$

$$g = 32.17 \text{ ft/sec}^2 = 386.04 \text{ in/sec}^2$$

$$Vol = \pi(b^2 - a^2)\ L$$

$$m = \left(\frac{0.282}{386.04}\right) Vol$$

(I)

$$b_I = \frac{4.36}{2} = 2.18$$

$$a_I = \frac{3.40}{2} = 1.70$$

$$L_I = 1.35$$

$$Vol_I = \pi(2.18^2 - 1.70^2)\ 1.35 = 7.8987$$

$$M_I = \frac{1}{2}\left(\frac{0.282}{386.04}\right) Vol_I = 0.002885$$

accounts for lugs

$$W_I = M_I\, g = 1.1137167 \text{ lb}$$

$$I_{22_I} = \frac{M_I}{2}(2.18^2 - 1.7^2) = 0.00268651$$

(II)

$$b_{II} = \frac{3.40}{2} = 1.7$$

$$a_{II} = \frac{1.26}{2} = 0.63$$

$$L_{II} = 1.09$$

$$Vol_{II} = \pi(1.7^2 - 0.63^2)\, 1.09 = 8.53721$$

$$M_{II} = \left(\frac{0.282}{386.04}\right) Vol = 0.006236$$

$$W_{II} = M_{II}\, g = 2.4075 \text{ lb}$$

$$I_{22_{II}} = \frac{M_{II}}{2}(1.7^2 - 0.63^2) = 0.007773$$

(III)

$$b_{III} = \frac{3.4}{2} = 1.7$$

$$a_{III} = \frac{2.052}{2} = 1.026$$

$$L_{III} = 7.53$$

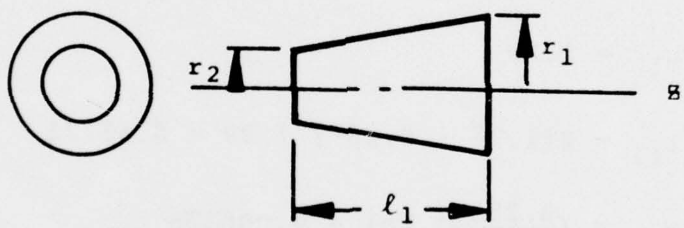
$$Vol_{III} = \Pi(1.7^2 - 1.026^2)\ 7.53 = 43.46409$$

$$M_{III} = \left(\frac{0.282}{386.04}\right) Vol_{III} = 0.031750$$

$$W_{III} = M_{III}\ g = 12.25687 \text{ lb}$$

$$I_{22_{III}} = \frac{M_{III}}{2}\ (1.7^2 - 1.026^2) = 0.0291675$$

(IV)



$$I_{22_1} = \frac{3}{10} M\ (r_1^2 - r_2^2)$$

$$r_1 = \frac{2.50}{2} = 1.25$$

$$r_2 = \frac{2.052}{2} = 1.026$$

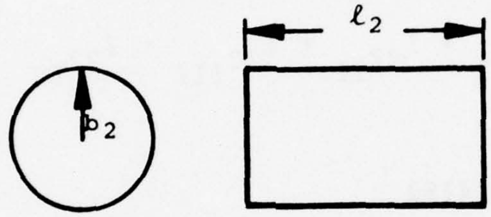
$$L_1 = 0.5$$

Using Prismoidal's formula we find

$$Vol_1 = \frac{\pi}{3} L (r_1^2 + r_1 r_2 + r_2^2) = 2.040818$$

$$M_1 = (\frac{0.282}{386.04}) Vol_1 = 0.0014908$$

$$W_1 = M_1 g = 0.5755 \text{ lb}$$



$$b_2 = \frac{3.40}{2} = 1.7$$

$$L_2 = 0.5$$

$$Vol_2 = \pi b_2^2 L_2 = 4.5396014$$

$$M_2 = (\frac{0.282}{386.04}) Vol_2 = 0.00331615$$

$$W_2 = M_2 g = 1.280168 \text{ lb}$$

$$I_{22_2} = \frac{1}{2} M_2 b_2^2$$

$$I_{22_{IV}} = I_{22_2} - I_{22_1}$$

$$I_{22_{IV}} = \frac{1}{2} M_2 \ (1.7)^2 - \frac{3}{10} M \ \ (1.25^2 - 1.026^2)$$

$$I_{22_{IV}} = 0.004792 - 0.000228 = 0.004564$$

$$I_{22} = I_{22_I} + I_{22_{II}} + I_{22_{III}} + I_{22_{IV}}$$

$$I_{22} = 0.044191$$

$$W = W_I + W_{II} + W_{III} + W_{IV}$$

$$W = 16.4828$$

OUTPUT DATA (TIME DEPENDENT)

1.	TIME	Time (sec)
2.	Q(1), q_1	Receiver displacement relative to turret (in)
3.	Q(2), q_2	Slide displacement relative to the receiver (in)
4.	QD(1), $\dot{q}_1$	Receiver velocity relative to turret (in/sec)
5.	QD(2), $\dot{q}_2$	Slide velocity relative to the receiver (in/sec)
6.	QDD(1), $\ddot{q}_1$	Receiver acceleration relative to the turret (in/sec^2)
7.	QDD(2), $\ddot{q}_2$	Slide acceleration relative to the receiver (in/sec^2)
8.	PC, P_c	Cylinder pressure (PSI)
9.	TC, T_c	Cylinder temperature (°R)
10.	TCD, $\dot{T}_c$	Time derivative of cylinder temperature (°R/sec)
11.	WO, W_o	Flow rate through orifice ($\frac{\text{lb-sec}}{\text{in}}$)
12.	PBL, P_{bl}	Barrel (receiver) pressure (PSI) (build up begins when projectile passes port)
13.	TBL, T_{bl}	Barrel (receiver) temperature (°R)
14.	XMACH, M_o	Mach number at throat (dimensionless)
15.	WOS	Total mass flow ($\int w_o dt$) ($\frac{\text{lb-sec}^2}{\text{in}}$)
16.	GASIMP	Total impulse applied to slide from cylinder ($\int P_c A_c dt$) (lb-sec)

17.	QC, q_c	Chamber displacement
18.	QCP	$\frac{dqc}{dq2}$ (dimensionless)
19.	QCPP	$\frac{d^2qc}{dq2^2}$ (in^{-1}
20.	QCD, $\dot{q}_c$	Velocity of the chamber relative to receiver (in/sec)
21.	THETC, θ_c	Rotation of the chamber (rad)
22.	THETCP	$(\frac{d\theta c}{dq_2})$ (rad/in)
23.	THEPP	$(\frac{d^2\theta c}{dq2^2})$ (rad/in^2)
24.	THETCD	Angular velocity of the chamber $(\frac{rad}{sec})$
25.	FR(1)	Recoil Force (lbs)
26.	FR(2)	Viscous damping action between receiver and turret (lbs)
27.	FR(3)	Forward buffer force of receiver (lbs)
28.	FR(4)	Rearward buffer force of receiver (lbs)
29.	FR(5)	-FS(1) (lbs)
30.	FR(6)	-FS(2) (lbs)
31.	FR(7)	-FS(3) (lbs)
32.	FR(8)	-FS(4) (lbs)
33.	FR(9)	-FS(5) (lbs)
34.	FR(10)	-FS(6) (lbs)
35.	FR(11)	-FS(7) (lbs)

36.	FR(12)	-FC(1) (lbs)
37.	FR(13)	-FC(2) (lbs)
38.	FR(14)	-FC(3) (lbs)
39.	FR(15)	Piece-wise constant dissipative force (coulomb friction) (lbs)
40.	FS(1)	Drive spring on slide (lbs)
41.	FS(2)	Viscous damping between receiver and slide (lbs)
42.	FS(3)	Cylinder pressure force (lbs)
43.	FS(4)	Slide rear buffer force (lbs)
44.	FS(5)	Sear spring force (lbs)
45.	FS(6)	Slide forward buffer force (lbs)
46.	FS(7)	Piece-wise constant dissipative force (lbs)
47.	FC(1)	Chamber rear buffer force (lbs)
48.	FC(2)	Viscous damping between receiver and chamber (lbs)
49.	FC(3)	Piece-wise constant dissipative force (lbs)
50.	FC(4)	Propellant force (lbs)
51.	VPØTQ(1)	Weight force ($\frac{\partial v}{\partial q_1}$) (lbs)
52.	VPØTQ(2)	Weight force ($\frac{\partial v}{\partial q_2}$) (lbs)
53.	ENERGY	Parameter computed to determine firing (in-lbs)
54.	FXR	Sum of forces acting on receiver (not weight or constraint forces)(lbs)

55.	FXS	Sum of forces acting on slide (not weight or constraint forces) (lbs)
56.	FXC	Sum of forces acting on chamber (not weight or constraint forces) (lbs)
57.	XMC(1)	Sum of moments acting on chamber (in-lbs)
58.	TXT	x-coordinate of turret (center of rotation) in inertial frame (m)
59.	TXTP	Velocity (x-coordinate) in inertial frame (m/sec)
60.	TXTPP	Acceleration (x-coordinate) in inertial frame (m/sec^2)
61.	TYT	y-coordinate of turret (center of rotation) in inertial frame (m)
62.	TYTP	Velocity (y-coordinate) in inertial frame (m/sec)
63.	TYTPP	Acceleration (y-coordinate) in inertial frame (m/sec^2)
64.	TZT	z-coordinate of turret (center of rotation) in inertial frame (m)
65.	TZTP	Velocity (z-coordinate) inertial frame (m/sec)
66.	TZTPP	Acceleration (z-coordinate) inertial frame (m/sec^2)
67.	PSI	Euler angle, Ψ, (rad)
68.	PSIP	$\dot{\Psi}$ (rad/sec)
69.	PSIPP	$\ddot{\Psi}$ (rad/sec^2)
70.	THETA	Elevation, θ, (rad)
71.	THETD	$\dot{\theta}$ (rad/sec)
72.	THEPP	$\ddot{\theta}$ (rad/sec^2)

OUTPUT DATA (TIME INDEPENDENT)

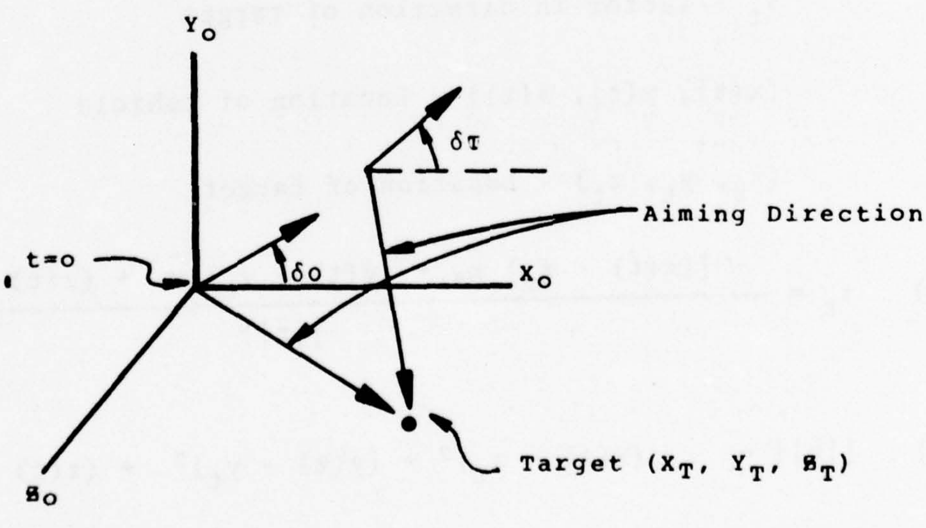
1.	THEOC, θ_{oc}	Angle associated with kinematic constraints (rad)
2.	YOCI, Y_{oci}	Distance associated with kinematic constraints (in)
3.	ACI, A_{ci}	Major axis of ellipse for kinematic constraint (in)
4.	XCI, X_{ci}	Distance associated with kinematic constraints (in)
5.	QCI, q_{ci}	Distance associated with kinematic constraint
6.	YOCF, Y_{ocf}	Distance associated with kinematic constraint (in)
7.	THEOF, θ_{ocf}	Angle associated with kinematic constraint (rad)
8.	ACF, a_{cf}	Distance associated with kinematic constraint (in)
9.	XCF, x_{cf}	Distance associated with kinematic constraint (in)
10.	YCF, y_{cf}	Distance associated with kinematic constraint (in)
11.	QCF, q_{cf}	Distance associated with kinematic constraint (in)
12.	YOQF, y_{oqf}	Distance associated with kinematic constraint (in)
13.	THEOQ, θ_{oq}	Angle associated with kinematic constraint (rad)
14.	AQF, a_{qf}	Distance associated with kinematic constraint (in)
15.	UQF, u_{qf}	Distance associated with kinematic constraint (in)

16.	QCO, q_{co}	Distance associated with kinematic constraint (in)
17.	RB, R_b	Gas constant $\left(\frac{in^2}{sec^2 - °R}\right)$
18.	GAMMAB, γ_b	Ratio of specific heats (dimensionless)
19.	RHØA, ρ_a	Density of air ($lbs\text{-}sec^2/in^4$)
20.	CRRAT	Critical pressure ratio (dimensionless)
21.	XMASS(I)	Mass of receiver, slide and chamber ($lbs\text{-}sec^2/in$)
22.	QREF, q_{ref}	Position of slide in equilibrium position (in)
23.	RIMPUL	Chamber impulse (lbs-sec) ($\int c(t)dt$)
24.	TOTMAS	Total mass of receiver, slide and chamber
25.	FRECO, $\frac{F_s+F_r}{2}$	Average recoil force (lbs)
26.	RECOLD, L	Nominal recoil distance (in)
27.	YDDOT, $\ddot{y}$	Acceleration of turret to correct for firing time (in/sec^2)

FLIGHT PATH CALCULATIONS

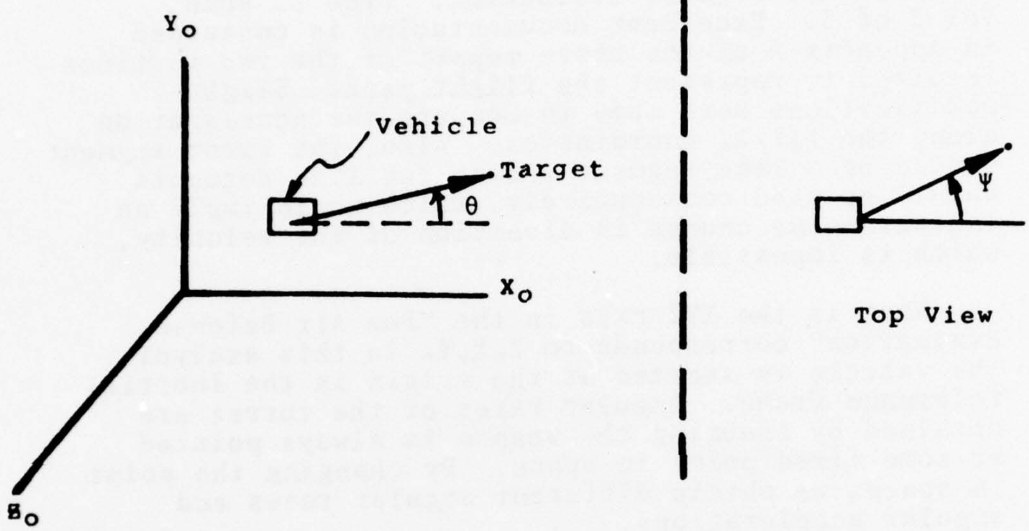
The flight path is represented by a series of linear, circular, and spiral segments. It uses the same flight path representation as contained in "The FUE Air Defense Evaluation," Fred L. Bunn, Vol 3 of 3. Excellent documentation is contained in Appendix A of the above report of the two routines required to represent the flight path. Slight modifications were made to compute the acceleration along the $X_oY_oZ_o$ coordinates. Also, the first segment should be a line segment and no two line segments should be used consecutively, which would imply an instantaneous change in direction of the velocity, which is impossible.

What is the XYZ axis in the "Fue Air Defense Evaluation" corresponds to $Z_oX_oY_o$ in this analysis. The vehicle is started at the origin in the inertial reference frame. Angular rates of the turret are obtained by assuming the weapon is always pointed at some fixed point in space. By changing the point in space, we obtain different angular rates and angular accelerations.



δ_o - Heading angle of aircraft at t = o

δ_t - Heading angle at a later time



τ_t - Vector in direction of target

(x(t), y(t), z(t)) - Location of vehicle

(x_t, y_t, z_t) - Location of target

(1.b) $$\tau_t = \frac{-\left[(x(t) - x_t)\,\hat{x}_o + (y(t) - y_t)\,\hat{y}_o + (z(t) - Z_t)\,\hat{z}_o\right]}{||\bar{r}||}$$

(2.b) $$||\bar{r}|| = (x(t) - x_t)^2 + (y(t) - y_t)^2 + (z(t) - z_t)^2$$

See main analysis for rotational transformation.

From geometry we have

(3.b) $$\tau_t = \text{Cos}\ \Psi\ \text{Cos}\ \theta\ \hat{x}_o + \text{Cos}\ \Psi\ \text{Sin}\ \theta\ \hat{y}_o - \text{Sin}\ \Psi\ \hat{z}_o$$

From equation (3.b)

(4.b) $$\tan\theta = \frac{y(t) - y_t}{x(t) - x_t}$$

$$\theta = \tan^{-1}\left(\frac{y(t) - y_t}{x(t) - x_t}\right)$$

(Use ATAN2 in computer)

(5.b) $$\tan\Psi = \frac{-\ (z(t) - z_t)\ \text{Sin}\ \theta}{(y(t) - y_t)}$$

From equation (4.b)

(6.b) $$\dot{\theta} = \frac{\text{Cos}^2\ \theta}{(x(t) - x_t)^2}\ [\dot{y}(t)\ (x(t) - x_t) - (y(t) - y_t)\ \dot{x}(t)]$$

From equation (5.b)

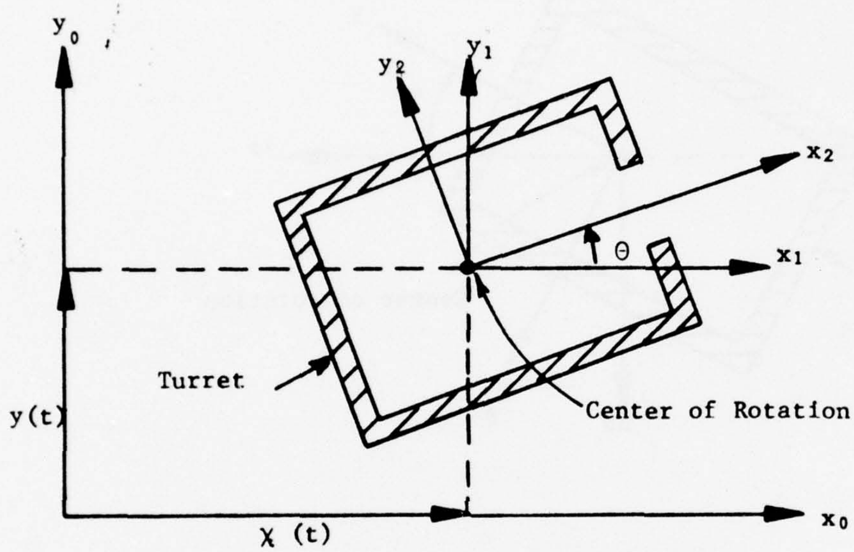
(7.b) $$\dot{\Psi} = -\ \frac{\text{Cos}^2\Psi}{(y(t) - y_t)^2}\ [(\dot{z}(t)\ \text{Sin}\ \theta + (z(t) - z_t)\ \dot{\theta}\ \text{Cos}\ \theta)$$

$$(y(t) - y_t) + (z(t) - z_t)\ \dot{y}(t)\ \text{Sin}\ \theta]$$

$$(8.b)\quad \ddot{\theta} = [\{-\dot{\theta} \operatorname{Cos}\theta \operatorname{Sin}\theta\, (\dot{y}(t)(x(t) - x_t) - (y(t) - y_t)\, \dot{x}(t)) + \operatorname{Cos}^2\theta\, (\ddot{y}(t)(x(t) - x_t) - (y(t) - y_t)\, \ddot{x}(t))\}\, (x(t) - x_t)^2 - 2 \operatorname{Cos}^2\theta\, (\dot{y}(t)(x(t) - x_t) - (y(t) - y_t)\, \dot{x}(t))\, \dot{x}(t)(x(t) - x_t)]/(x(t) - x_t)^4$$

$$(9.b)\quad \ddot{\Psi} = -[\{-\dot{\Psi} \operatorname{Cos}\Psi \operatorname{Sin}\Psi\, ((\dot{z}(t) \operatorname{Sin}\theta + (z(t) - z_t)\, \dot{\theta} \operatorname{Cos}\theta)\, (y(t) - y_t) + (z(t) - z_t)\, \dot{y}(t) \operatorname{Sin}\theta) + \operatorname{Cos}^2\Psi\, [\dot{y}(t)(\dot{z}(t) \operatorname{Sin}\theta + (z(t) - z_t)\, \dot{\theta} \operatorname{Cos}\theta) + (y(t) - y_t)(\ddot{z}(t) \operatorname{Sin}\theta + \dot{z}(t)\, \dot{\theta} \operatorname{Cos}\theta + \dot{z}(t)\, \dot{\theta} \operatorname{Cos}\theta - \dot{\theta}^2 \operatorname{Sin}\theta\, (z(t) - z_t) + (z(t) - z_t)\ddot{\theta} \operatorname{Cos}\theta) + \dot{z}(t)\, \dot{y}(t) \operatorname{Sin}\theta + (z(t) - z_t)\, \dot{\theta}\, \dot{y}(t) \operatorname{Cos}\theta + (z(t) - z_t)\, \ddot{y}(t) \operatorname{Sin}\theta]\}\, (y(t) - y_t)^2 - 2\dot{y}(t) \operatorname{Cos}^2\Psi\, (y(t) - y_t)\, [(\dot{z}(t) \operatorname{Sin}\theta + (z(t)-z_t)\, \dot{\theta} \operatorname{Cos}\theta)\, (y(t) - y_t) + (z(t) - z_t)\, \dot{y}(t) \operatorname{Sin}\theta]]\, /\, (y(t) - y_t)^4$$

Side View

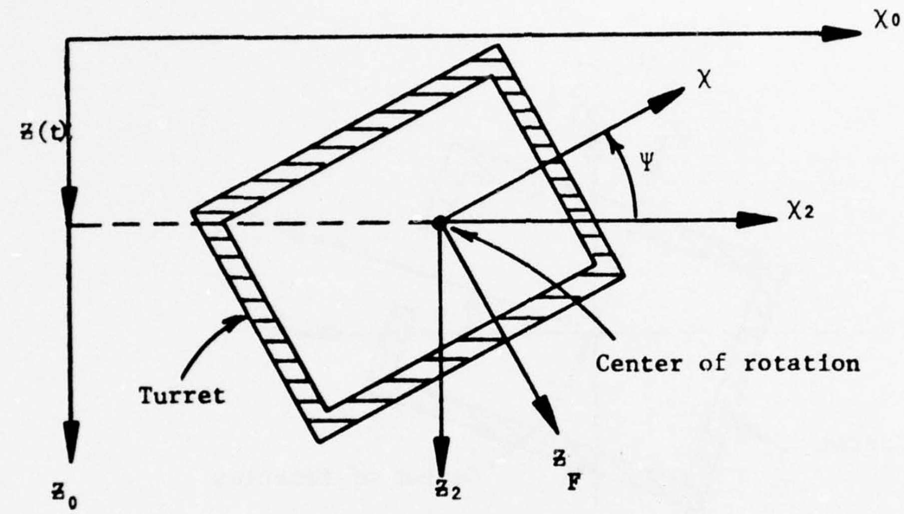


x_o, y_o, z_o - Inertial reference frame

x_1, y_1, z_1 - Origin at center of rotation (parallel with x_o, y_o, z_o)

x_2, y_2, z_2 - Origin at center of rotation (rotated about z, through angle θ)

Top View



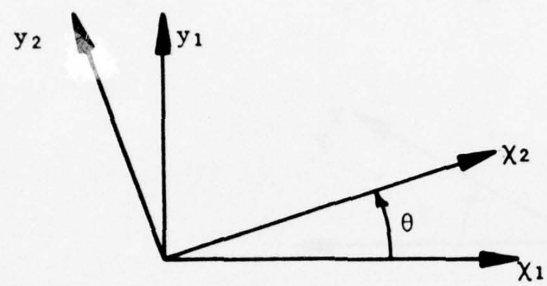
x_f, y_f, z_f - Fixed body axis (rotated about y_2, through angle Ψ)

The center of rotation is allowed to move in space; position is given by:

$$\bar{r}_{cg} = x(t)\ \hat{x}_o + y(t)\ \hat{y}_o + z(t)\ \hat{z}_o$$

Also, the two "Euler Angles," θ and Ψ, are described as functions of time (the first derivatives represent slewing rates).

We now derive transformation relationship between coordinate systems:



(1) $$\hat{x}_2 = \text{Cos}\ \theta\ \hat{x}_1 + \text{Sin}\ \theta\ \hat{y}_1$$

$$\hat{y}_2 = -\text{Sin}\ \theta\ \hat{x}_1 + \text{Cos}\ \theta\ \hat{y}_1$$

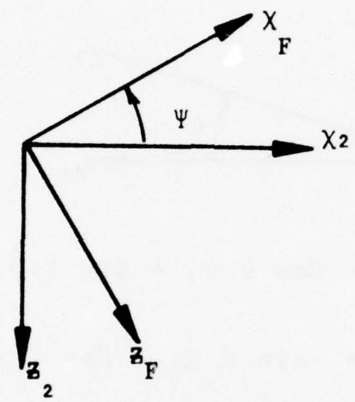
$$\hat{z}_2 = \hat{z}_1$$

where $\hat{x}_2$ is a unit vector along x_2 axis.

Let $$\overline{X}_i = \begin{bmatrix} \hat{x}_i \\ \hat{y}_i \\ \hat{z}_i \end{bmatrix}$$ (column matrix)

Then equation (1) can be written as:

(2) $$\overline{X}_2 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \overline{X}_1$$



(3) $$\hat{x}_f = \cos\Psi\,\hat{x}_2 - \sin\Psi\,\hat{z}_2$$

$$\hat{y}_f = \hat{y}_2$$

$$\hat{z}_f = \sin\Psi\,\hat{x}_2 + \cos\Psi\,\hat{z}_2$$

Equation (3) can be rewritten as,

(4) $$\overline{X}_f = \begin{bmatrix} \cos\Psi & 0 & -\sin\Psi \\ 0 & 1 & 0 \\ \sin\Psi & 0 & \cos\Psi \end{bmatrix} \overline{X}_2$$

From equations (2) and (4),

$$(5)\qquad \overline{X}_f = \begin{bmatrix} \cos\Psi & 0 & -\sin\Psi \\ 0 & 1 & 0 \\ \sin\Psi & 0 & \cos\Psi \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \overline{X}_1$$

$$(6)\qquad \overline{X}_f = \begin{bmatrix} \cos\Psi\cos\theta & \cos\Psi\sin\theta & -\sin\Psi \\ -\sin\theta & \cos\theta & 0 \\ \sin\Psi\cos\theta & \sin\Psi\cos\theta & \cos\Psi \end{bmatrix} \overline{X}_1$$

We now determine the "kinematics" of the receiver. We first determine the angular velocity and, secondly, we determine the C.G. relative to the inertial frame of reference. (Angular velocity is determined relative to the fixed body axis, x_f, y_f, z_f)

The angular velocity of the receiver is given by,

$$(7)\qquad \overline{W}_{m_r} = \dot{\theta}\,\hat{z}_1 + \dot{\Psi}\,\hat{y}_2$$

$\overline{X}_f$ is of the form,

$$(8)\qquad \overline{X}_f = A\,\overline{X}_1$$

and from equation (8),

$$(9)\qquad \overline{X}_1 = A^{-1}\,\overline{X}_f = A^t\overline{X}_f$$

From equations (6) and (9) we have,

$$\overline{X}_1 = \begin{bmatrix} \text{Cos } \Psi \text{ Cos } \theta & -\text{Sin } \theta & \text{Sin } \Psi \text{ Cos } \theta \\ \text{Cos } \Psi \text{ Sin } \theta & \text{Cos } \theta & \text{Sin } \Psi \text{ Sin } \theta \\ -\text{Sin } \Psi & 0 & \text{Cos } \Psi \end{bmatrix} \overline{X}_f \tag{10}$$

From equation (1) it can be seen that,

$$\hat{z}_1 = -\text{Sin } \Psi \, \hat{x}_f + \text{Cos } \Psi \, \hat{z}_f \tag{11}$$

From equation (4) we have,

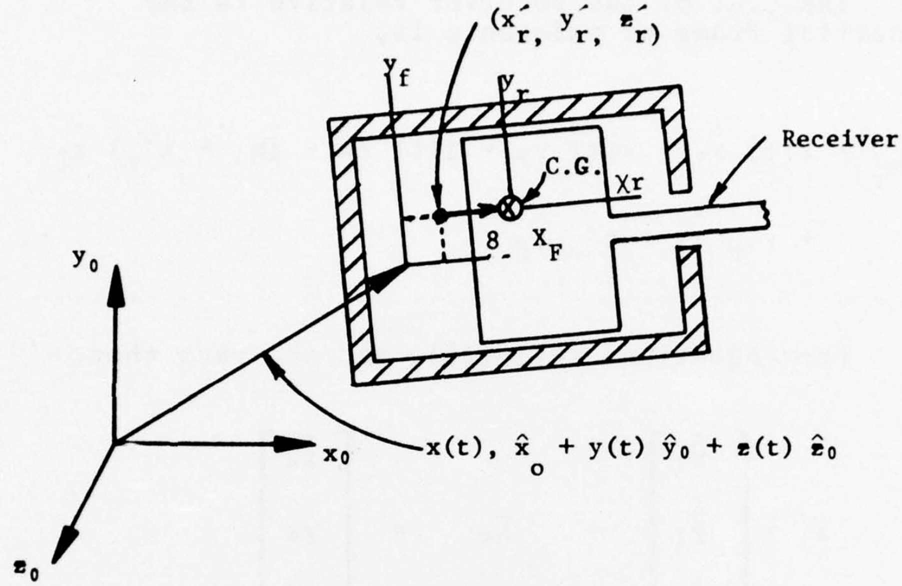
$$\overline{X}_2 = \begin{bmatrix} \text{Cos } \Psi & 0 & \text{Sin } \Psi \\ 0 & 1 & 0 \\ -\text{Sin } \Psi & 0 & \text{Cos } \Psi \end{bmatrix} \overline{X}_f \tag{12}$$

From equation (12),

$$\hat{y}_2 = \hat{y}_f \tag{13}$$

From equations (7), (11), and (12) we have,

$$\overline{W}_{m_r} = -\dot{\theta} \text{ Sin } \Psi \, \hat{x}_f + \dot{\Psi} \, \hat{y}_f + \dot{\theta} \text{ Cos } \Psi \, \hat{z}_f \tag{14}$$



(x'_r, y'_r, z'_r) - Initial position relative to x_f, y_f, z_f coordinate system.

x_r, y_r, z_r - Fixed body axis of the receiver. (x_r is parallel with translation axis)

q_1 - Travel along the translation axis of the C.G. of the receiver.

The C.G. of the receiver relative to the inertial frame of reference is,

$$(15) \qquad \bar{r}_{m_r} = x(t)\,\hat{x}_o + y(t)\,\hat{y}_o + z(t)\,\hat{z}_o + (q_1 + x'_r)\,\hat{x}_f + y'_r\,\hat{y}_f + z'_r\,\hat{z}_f$$

From equations (6), (15), and the fact that,

$$(16) \qquad \bar{X}_1 = \begin{bmatrix} \hat{x}_1 \\ \hat{y}_1 \\ \hat{z}_1 \end{bmatrix} = \bar{X}_o = \begin{bmatrix} \hat{x}_o \\ \hat{y}_o \\ \hat{z}_o \end{bmatrix}$$

$$(17) \qquad \bar{r}_{m_r} = \{x(t) + (q_1 + x'_r)\,\text{Cos}\,\Psi\,\text{Cos}\,\theta - y'_r\,\text{Sin}\,\theta + z'_r\,\text{Sin}\,\Psi\,\text{Cos}\,\theta\}\,\hat{x}_o$$

$$+ \{y(t) + (q_1 + x'_r)\,\text{Cos}\,\Psi\,\text{Sin}\,\theta + y'_r\,\text{Cos}\,\theta + z'_r\,\text{Sin}\,\Psi\,\text{Sin}\,\theta\}\,\hat{y}_o$$

$$+ \{z(t) - (q_1 + x'_r)\,\text{Sin}\,\Psi + z'_r\,\text{Cos}\,\Psi\}\,\hat{z}_o$$

Also, from the fact that,

$$\overline{X}_f = \overline{X}_r \tag{18}$$

we can rewrite equation (14) as,

$$\overline{W}_{m_r} = -\dot{\theta} \text{ Sin } \Psi \; \hat{x}_r + \dot{\Psi} \; \hat{y}_r + \dot{\theta} \text{ Cos } \Psi \; \hat{z}_r \tag{19}$$

The computer operates on equations (17) and (19) to generate equations of motion; see part A.

Since $\overline{W}_{m_r}$ is independent of the degrees of freedom, the equations of motion will be independent of $\overline{W}_{m_r}$. Therefore, orientation of x_r, y_r, z_r is not important for determining moments of inertia, hence we do not have to calculate them.

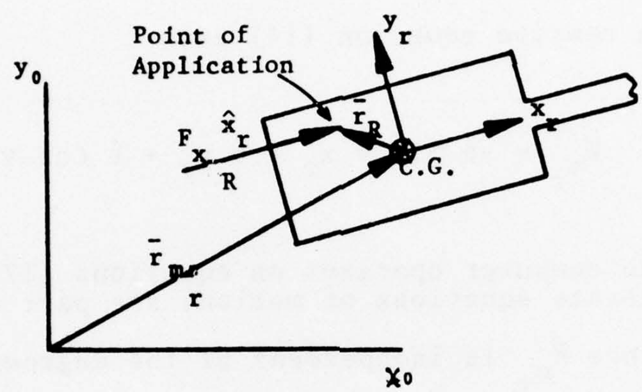
Equation (17) gives the C.G. of the receiver relative to the inertial reference frame. Equation (19) gives the angular velocity of the receiver relative to the x_r, y_r, z_r coordinates. The coordinate q_1 must be determined yet.

We consider forces applied to the receiver along the translational axis as,

$$\overline{F}_r = F_{x_r} \; \hat{x}_r \tag{20}$$

The point of application is given by,

(21)
$$\bar{r}_r = \bar{r}_{m_r} + \bar{r}'_r$$



From equations (15) and (18) we have,

(22)
$$\bar{r}_{m_r} = x(t)\ \hat{x}_o + y(t)\ \hat{y}_o + z(t)\ \hat{z}_o$$

$$+ (q_1 + x'_r)\ \hat{x}_r + y'_r\ \hat{y}_r + z'_r\ \hat{z}_r$$

The generalized force equation is,

(23)
$$Q_{q_i} = \frac{\partial \bar{r}_r}{\partial q_i} \cdot \bar{F}_r$$

if $i \neq 1$ $\quad \frac{\partial \bar{r}_r}{\partial q_i} = 0$

if $i = 1$ from equation (22),

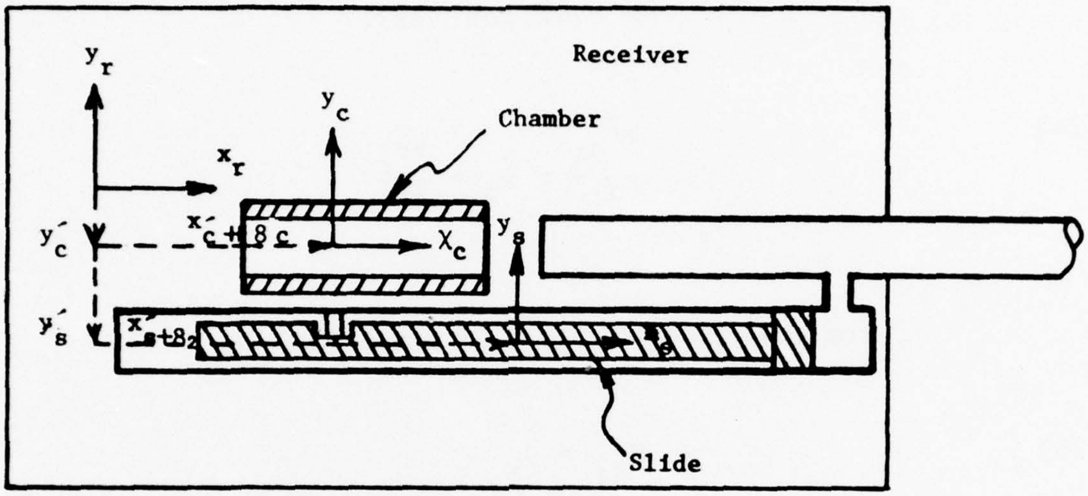
(24) $$\frac{\partial \bar{r}_{m_r}}{\partial q_1} = \hat{x}_r = \frac{\partial \bar{r}_r}{\partial q_1}$$

From equations (20), (23), and (24)

(25) $$Q_{q_1} = F_{x_r}$$

F_{x_r} includes all forces on receiver (except gravitational and constraint forces) along translation axis.

Dynamics of Internal Parts



M_s	mass of slide
M_c	mass of chamber
$x_c\ y_c\ z_c$	coordinate system fixed to the chamber (coincides with principal axis)
$(x'_s, \dot{y}'_s, z'_s)$	slide position at t = 0 relative to $x_r\ y_r\ z_r$ coordinate system
(x'_c, y'_c, z'_c)	chamber position at t = 0 relative to $x_r\ y_r\ z_r$ coordinate system

Note: $\overline{x_s} = \overline{x_r}$

The angular velocity of the slide is given by:

$$\overline{W}_{m_s} = \overline{W}_{m_r} = -\dot{\theta}\ \mathrm{Sin}\ \psi\ \hat{x}_s + \dot{\psi}\ \hat{y}_s + \dot{\theta}\ \mathrm{Cos}\ \psi\ \hat{z}_s \tag{26}$$

The C.G. of the slide relative to the inertial frame of reference is,

$$\bar{r}_{m_s} = \bar{r}_{m_r} + (x'_s + q\ )\ \hat{x}_r + y'_s\ \hat{y}_r + z'_s\ \hat{z}_r \tag{27}$$

From equations (15) and (18) we can rewrite equation (27) as:

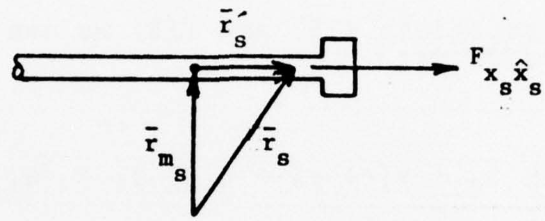
$$\bar{r}_{m_s} = x(t)\ \hat{x}_\circ + y(t)\ \hat{y}_\circ + z(t)\ \hat{z}_\circ + (q_1 + q_2 + x'_s + x'_r)\ \hat{x}_f + (y'_s + y'_r)\ \hat{y}_f + (z'_r + z'_s)\ \hat{z}_f \tag{28}$$

From equations (6) and (16) we can rewrite equation (28) as:

$$\begin{aligned}\bar{r}_{m_s} = {} & [x(t) + (q_1 + q_2 + x'_x + x'_r)\ \mathrm{Cos}\ \psi\ \mathrm{Cos}\ \theta \\ & - (y'_s + y'_r)\ \mathrm{Sin}\ \theta + (z'_r + z'_s)\ \mathrm{Sin}\ \psi\ \mathrm{Cos}\ \theta]\ \hat{x}_\circ \\ & + [y(t) + (q_1 + q_2 + x'_s + x'_r)\ \mathrm{Cos}\ \psi\ \mathrm{Sin}\ \theta \\ & + (y'_s + y'_r)\ \mathrm{Cos}\ \theta + (z'_r + z'_s)\ \mathrm{Sin}\ \psi\ \mathrm{Sin}\ \theta]\ \hat{y}_\circ \\ & + [z(t) - (q_1 + q_2 + x'_s + x'_r)\ \mathrm{Sin}\ \psi \\ & + (z'_r + z'_s)\ \mathrm{Cos}\ \psi]\ \hat{z}_\circ\end{aligned} \tag{29}$$

The computer operates on equations (26) and (29) to obtain expressions for equations of motion.

We shall be interested in the generalized forces from the force applied to the slide along the translational axis at the point $\overline{r}_s$.



$$\overline{r}_s = \overline{r}_{m_s} + \overline{r}'_s$$

(r'_s is independent of q_1 or q_2)

$$Q_1 = \frac{\partial \overline{r}_s}{\partial q_1} \cdot F_{x_s} \hat{x}_s \qquad (30)$$

$$\frac{\partial \overline{r}_s}{\partial q_1} = \frac{\partial \overline{r}_{m_s}}{\partial q_1}$$

From equation (28) and the fact that $\hat{x}_f = \hat{x}_s$

$$Q_1 = F_{x_s}$$

Also,

$$Q_2 = \frac{\partial \bar{r}_{m_s}}{\partial q_2} \cdot F_{x_s} \hat{x}_s = F_{x_s} \tag{31}$$

From equations (30) and (31), the generalized forces are independent of the point of application on the slide.

We now obtain the kinematics of the chamber. The rotation of the chamber is a function of q_2 which we call $\theta_c (q_2)$. The translation of the chamber has a certain amount of motion relative to the slide when the weapon is unlocked, but we will not add another degree of freedom to account for this relative motion.

The location of the C.G. of the chamber is given by:

$$\bar{r}_{m_c} = \bar{r}_{m_r} + (x'_c + q_c)\, \hat{x}_r + y'_c\, \hat{y}_r + z'_c\, \hat{z}_r \tag{32}$$

(q_c is a function of q_2)

Equation (32) is of the same form as equation (27) with q_2 replaced by q_c and x'_s, y'_s, z'_s replaced by x'_c, y'_c, z'_c respectively. Therefore, from equation (29) we can rewrite equation (32) as:

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(33) $$\bar{r}_{m_c} = [x(t) + (q_1 + q_c + x'_c + x'_r) \cos \Psi \cos \theta$$

$$- (y'_c + y'_r) \sin \theta + (z'_r + z'_c)$$

$$\sin \Psi \cos \theta] \hat{x}_o$$

$$+ [y(t) + (q_1 + q_c + x'_c + x'_r) \cos \Psi \sin \theta$$

$$+ (y'_c + y'_r) \cos \theta + (z'_r + z'_c)$$

$$\sin \Psi \sin \theta] \hat{y}_o$$

$$+ [z(t) - (q_1 + q_c + x'_c + x'_r) \sin \Psi$$

$$+ (z'_r + z'_c) \cos \Psi] \hat{z}_o$$

For forces acting along the translation axis on the chamber, the generalized forces are given by,

(34) $$Q_1 = \frac{\partial \bar{r}_s}{\partial q_1} \cdot F_{x_c} \hat{x}_c = F_{x_c}$$

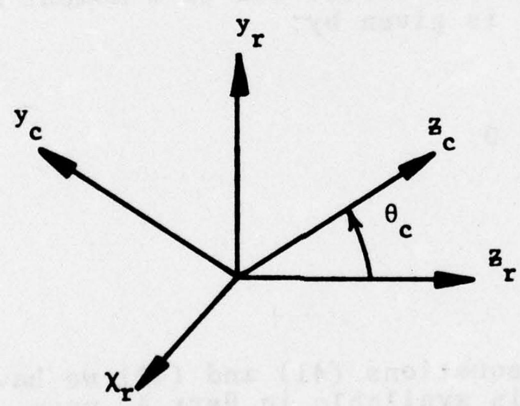
(35) $$Q_2 = \frac{\partial \bar{r}_s}{\partial q_2} \cdot F_{x_c} \hat{x}_c = F_{x_c} \frac{\partial q_c}{\partial q_2}$$

The angular velocity of the chamber is given by,

$$\overline{W}_{m_c} = \overline{W}_{m_r} + \dot{\theta}_c \ (q_2) \ \hat{x}_c \tag{36}$$

From equation (19) we can rewrite equation (36) as,

$$\overline{W}_{m_c} = -\dot{\theta} \text{ Sin } \psi \ \hat{x}_r + \dot{\psi} \ \hat{y}_r + \dot{\theta} \text{ Cos } \psi \ \hat{z}_r + \dot{\theta}_c \ (q_2) \ \hat{x}_c \tag{37}$$



$$\begin{aligned} \hat{x}_r &= \hat{x}_c \\ \hat{y}_r &= \text{Cos } \theta_c \ \hat{y}_c + \text{Sin } \theta_c \ \hat{z}_c \\ \hat{z}_r &= -\text{Sin } \theta_c \ \hat{y}_c + \text{Cos } \theta_c \ \hat{z}_c \end{aligned} \tag{38}$$

From equations (37) and (38) we have,

$$(39) \qquad \overline{W}_{m_c} = [\dot{\theta}_c\ (q_2) - \dot{\theta}\ \text{Sin}\ \Psi]\ \hat{x}_c$$

$$+ [\Psi\ \text{Cos}\ \theta_c - \dot{\theta}\ \text{Cos}\ \Psi\ \text{Sin}\ \theta_c]\ \hat{y}_c$$

$$+ [\dot{\Psi}\ \text{Sin}\ \theta_c + \dot{\theta}\ \text{Cos}\ \Psi\ \text{Cos}\ \theta_c]\ \hat{z}_c$$

$$(40) \qquad \dot{\theta}_c = \frac{\partial \theta c}{\partial q_2}\ \dot{q}_2$$

The generalized forces due to a moment M_c applied to the chamber is given by:

$$(41) \qquad Q_1 = M_c\ \frac{\partial \theta c}{\partial q_1} = 0$$

$$(42) \qquad Q_2 = M_c\ \frac{\partial \theta c}{\partial q_2}$$

To derive equations (41) and (42) we have applied a result that is available in Part A, page

The gravitational potential is given by:

$$V = M_r g\ (\overline{r}_{m_r})_{y\text{-component}} + M_c g\ (\overline{r}_{m_c})_{y\text{-component}}$$

$$+ M_s g\ (\overline{r}_{m_s})_{y\text{-component}}$$

With no vehicle motion:

$x_t\ y_t\ z_t$ - coordinate system in direction of translation axis

Kinematic relationships are:

$$\bar{r}_{m_r} = (q_1 + x'_r)\,\hat{x}_t + y'_r\,\hat{x}_t + z'_r\,\hat{z}_t$$

$$\bar{W}_{m_r} = 0$$

$$\bar{r}_{m_s} = (q_1 + q_c + x'_r + x'_c)\,\hat{x}_t + (y'_r + y_c)\,\hat{y}_t$$

$$+ (z'_r + z'_c)\,\hat{z}_t$$

$$\bar{W}_{m_c} = \dot{\theta}_c\,(q_2)\,\hat{x}_c$$

$$V = M_r\, g \,\mathrm{Sin}\,\theta\,(q_1 + x'_r)$$

$$+ M_s\, g\,\mathrm{Sin}\,\theta\,(q_1 + q_2 + x'_r + x'_s)$$

$$+ M_c\, g\,\mathrm{Sin}\,\theta\,(q_1 + q_c + x'_r + x'_c)$$

(Generalized forces are the same as the general analysis.)

Locking and Unlocking Kinematics

To avoid discontinuous changes in velocities, we approximate all "cam constraint functions" by functions with continuous derivatives (see receiver and slide drawings for nominal values).

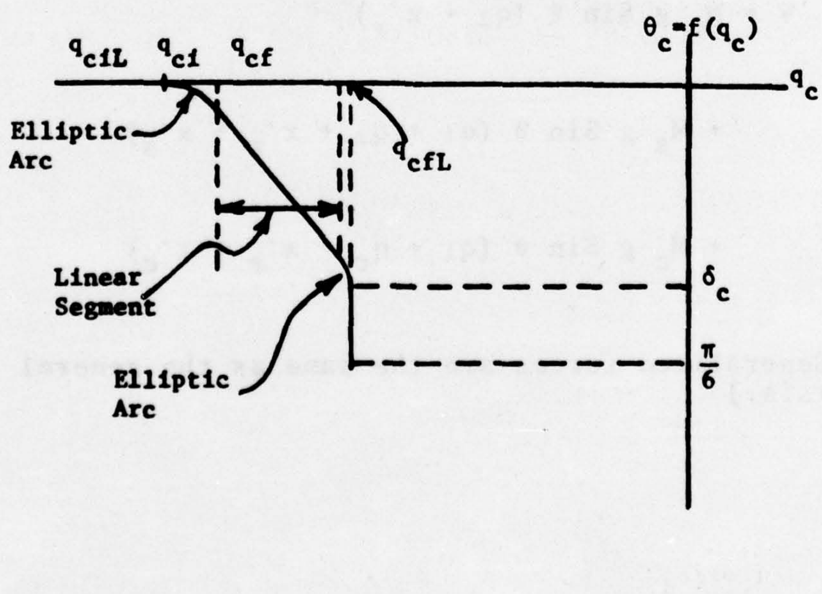
The rotation of the chamber is given by the following as a function of the displacement of the chamber position relative to the receiver.

q_{ciL} - value of q_c when locking begins

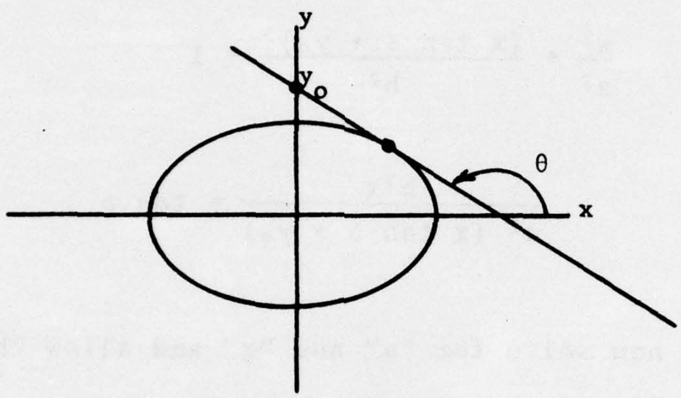
q_{cfL} - value of q_c when weapon is locked

q_{ci} - location of end of linear segment

q_{cf} - location of start of linear segment



We choose ellipses at the end of the linear segments to obtain continuous derivatives.



The equation of the line above is given by

(1) $$y = x \tan \theta + y_o$$

The general equation for an ellipses is,

(2) $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

From equations (1) and (2) we have,

(3) $$\frac{dy}{dx} = \frac{b^2 x}{a^2 y} = \tan \theta$$

From equations (1), (2), and (3) the following equations must be satisfied,

(4) $$\frac{x^2}{a^2} + \frac{(x \tan \theta + y_\circ)^2}{b^2} = 1$$

(5) $$\frac{-b^2 x}{a^2 (x \tan \theta + y_\circ)} = \tan \theta$$

We now solve for "a" and "x" and allow "b" to vary.

From equation (5) we have,

(6) $$\left(\frac{b^2}{a^2} + \tan^2 \theta\right) x = -y_\circ \tan \theta$$

(7) $$x = \frac{-y_\circ \tan \theta}{\frac{b^2}{a^2} + \tan \theta}$$

Let

(8) $$s = \frac{1}{a^2}$$

Substituting equations (7) and (8) into (4) we get,

(9) $$s \left(\frac{-y_\circ \tan \theta}{sb^2 + \tan^2 \theta}\right)^2 + \frac{1}{b^2} \left(\frac{-y_\circ \tan^2 \theta}{sb^2 + \tan^2 \theta} + y_\circ\right)^2 = 1$$

(10) $$S(-y_o \tan\theta)^2 + \frac{1}{b^2}\left[-y_o \tan^2\theta + y_o\,(sb^2 + \tan^2\theta)\right]^2$$

$$= (sb^2 + \tan^2\theta)^2$$

(11) $$S(-y_o \tan\theta)^2 + \frac{(y_o\, sb^2)^2}{b^2} = s^2b^4 + 2sb^2 \tan^2\theta + \tan^4\theta$$

(12) $$S^2(b^4 - y_o{}^2b^2) + S[2b^2 \tan^2\theta - (y_o \tan\theta)^2]$$

$$+ \tan^4\theta = 0$$

(13) $$S = \frac{-[2b^2 \tan^2\theta - (y_o \tan\theta)^2]}{2\,(b^4 - y_o{}^2b^2)}$$

$$\frac{\pm\ [2b^2 \tan^2\theta \quad (y_o \tan\theta)^2]^2 - 4(b^4\ y_o{}^2b^2)\tan^4\theta}{2\,(b^4 - y_o{}^2b^2)}$$

(14) $$S = \frac{\tan^2\theta}{y_o{}^2 - b^2}\ , \qquad \frac{-\tan^2\theta}{b^2}$$

To make "S" positive we use the minus sign for the discriminat in equation (13), thus

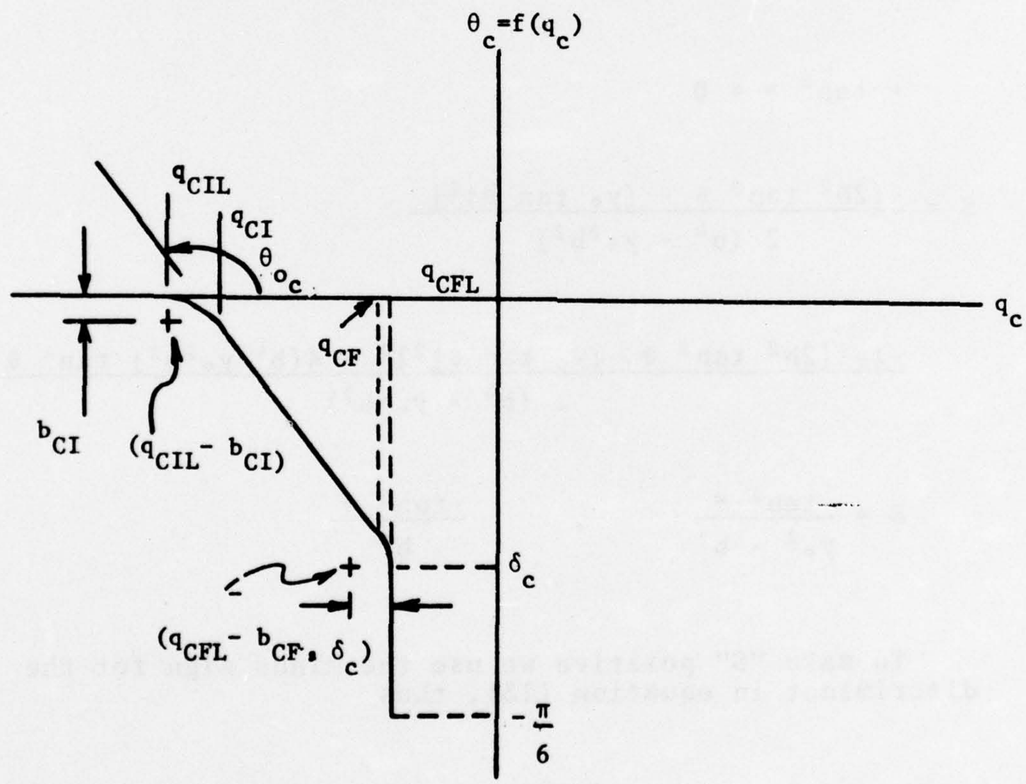
(14a) $$S = \frac{\tan^2\theta}{y_o{}^2 - b^2}$$

From equation (8),

$$a = \frac{1}{S} \tag{14b}$$

Using equations (14a) and (14b) the computation of "a" and "b" is arbitrary.

Applying the results from pages

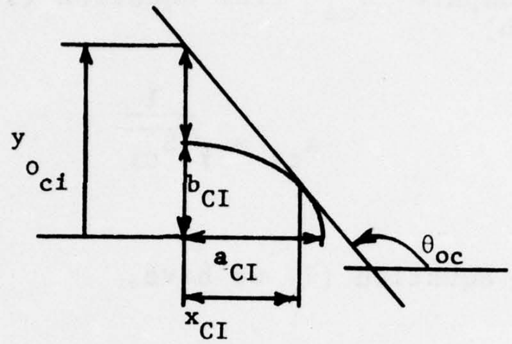


Nominal values:

$\delta_c = -\frac{\pi}{8}$

$q_{CFL} = 0$

$q_{CIL} = 1.02$ in



(15) $\theta_{\circ_C} = \tan^{-1} \left(\frac{-\delta_C}{q_{cfL} - q_{ciL}}\right)^{-1} + \frac{\pi}{2}$

(16) $y_{\circ_{ci}} = b_{ci} + \varepsilon_{ci}$

Nominal values:

ε_{ci} = 0.05 rad (chosen arbitrarily small to represent sharp corners)

b_{ci} = 0.0392 rad

From equations (14a) and (14b) with,

(17) $b = b_{ci}$

$y_\circ = y_{\circ_{ci}}$

$\theta = \theta_{\circ_C}$

We compute "S_{ci}" from equation (14a) and "a_{ci}" from (14b).

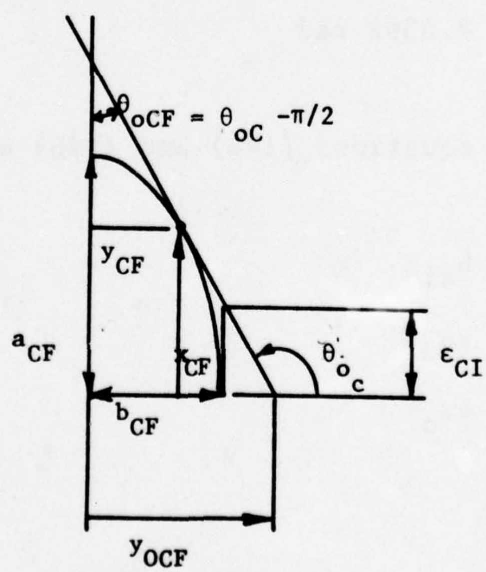
(18) $$a_{ci} = \sqrt{\frac{1}{S_{ci}}}$$

From equation (7) we have,

(19) $$x_{ci} = \frac{-y_{\circ ci} \tan \theta_{\circ c}}{\frac{b_{ci}{}^2}{a^2{}_{ci}} + \tan^2 \theta_{\circ c}}$$

(20) $$q_{ci} = q_{ciL} + x_{ci}$$

We will now look at the path prior to being fully locked.



(21) $$y_{ocf} = -\left(b_{cf} - \frac{\varepsilon_{ci}}{\tan \theta_{\circ c}}\right)$$

From equations (14a) and (14b) with,

(22) $$b = b_{cf}$$

$$y_\circ = y_{\circ cf} = -\left(b_{cf} - \frac{\varepsilon_{ci}}{\tan \theta_{\circ c}}\right)$$

$$\theta = \theta_{\circ cf} = \theta_{\circ c} - \frac{\pi}{2}$$

We compute "S_{cf}" from equation (14a) and "a_{cf}" from (14b)

(23) $$a_{cf} = \sqrt{\frac{1}{S_{cf}}}$$

Nominal values:

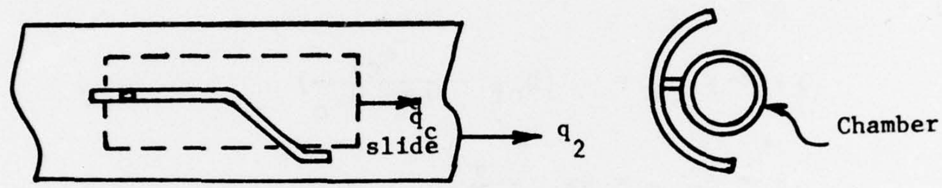
$b_{cf} = 0.10$ in

From equation (7) we have,

(24) $$x_{cf} = \frac{-\,y_{\circ cf} \tan \theta_{\circ cf}}{\dfrac{b_{cf}^2}{a_{cf}^2} + \tan^2 \theta_{\circ cf}}$$

$$y_{cf} = - \left[b^2_{cf} - \left(\frac{b_{cf}}{a_{cf}}\right)^2 x^2_{cf} \right]$$

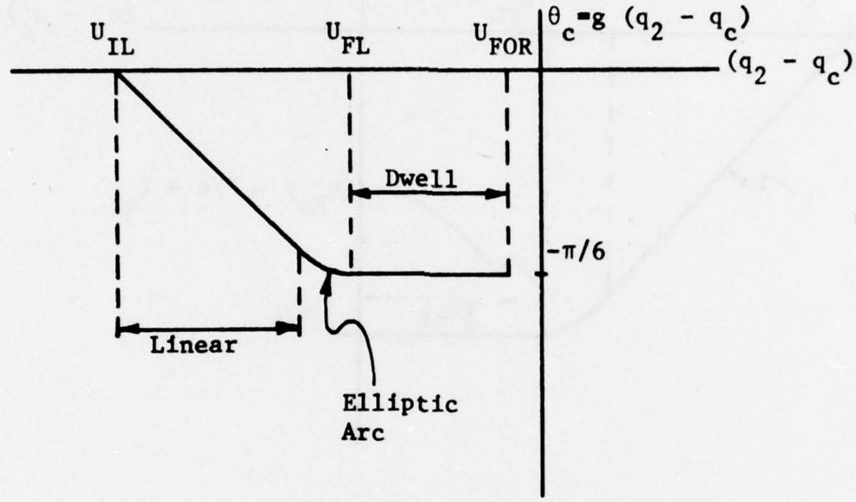
$$q_{cf} = q_{cfL} - (b_{cf} + y_{cf})$$



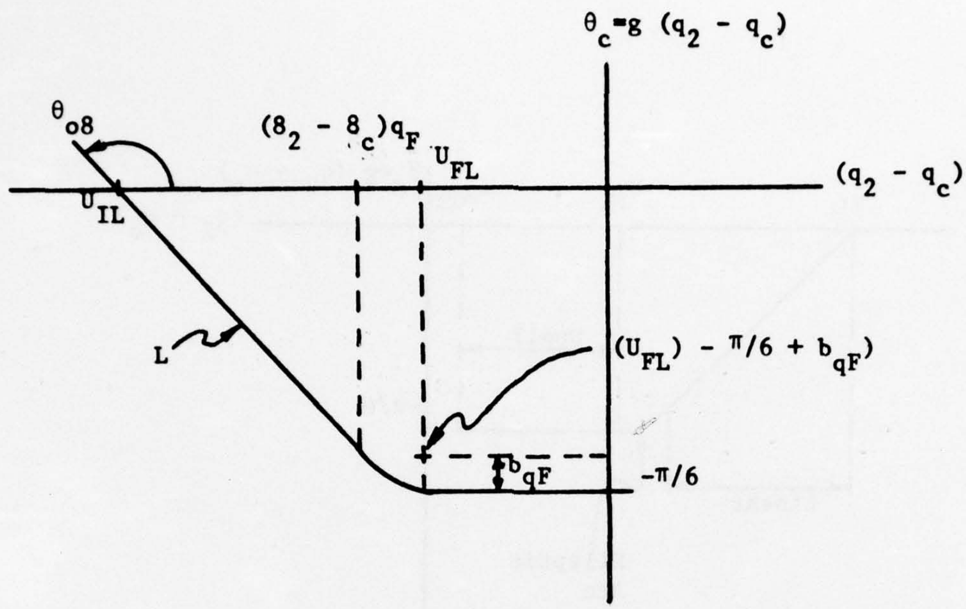
$U_{FOR} = (q_2 - q_c)_{FOR}$ - value of $(q_2 - q_c)$ when slide is fully forward.

$U_{FL} = (q_2 - q_c)_{FL}$ - value of $(q_2 - q_c)$ when chamber is fully loaded.

$U_{IL} = (q_2 - q_c)_{IL}$ - value of $(q_2 - q_c)$ when chamber is starting to lock.

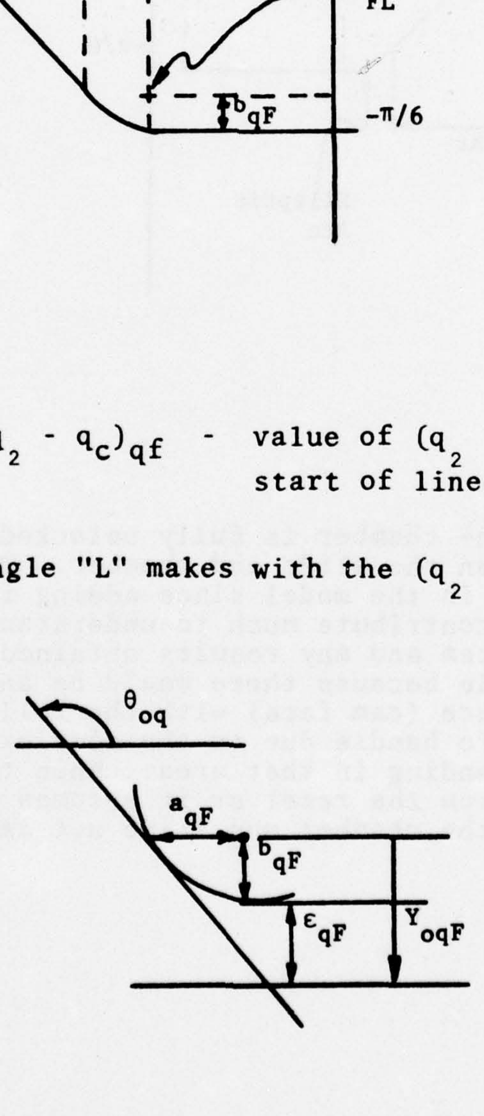


When the chamber is fully unlocked there is some play between the slide and chamber. This is not considered in the model since adding it to the model would not contribute much to understanding the response of the system and any results obtained would be questionable because there would be an impact of a metal surface (cam face) with the roller which is difficult to handle due to the complexity and lack of understanding in that area. When the chamber is unlocked from the receiver it becomes locked to the slide and the chamber and slide act as a single rigid body.



$U_{qf} = (q_2 - q_c)_{qf}$ - value of $(q_2 - q_c)$ at the start of linear segment

θ_{oq} - angle "L" makes with the $(q_2 - q_c)$ axis.



The values of ε_{qf} and b_{qf} are arbitrarily chosen to provide a sharp corner.

From equations (14a) and (14b) with,

$$Y_{\circ} = Y_{\circ qf} = -(b_{qf} + \varepsilon_{qf})$$

(25) $$\theta = \theta_{\circ q} = \tan^{-1}\left(\frac{\frac{\pi}{6} + \varepsilon_{qf}}{U_{fL} - U_{IL}}\right) + \frac{\pi}{2}$$

$$b = b_{qf}$$

we can compute "S_{qf}" and "a_{qf}".

Nominal values:

$$b_{qf} = 0.0344 \text{ rad}$$

$$\varepsilon_{qf} = 0.09 \text{ rad}$$

$$U_{FOR} = (q_2 - q_c)_{FOR} = 0.0$$

$$U_{FL} = (q_2 - q_c)_{FL} = +1.25 \text{ in}$$

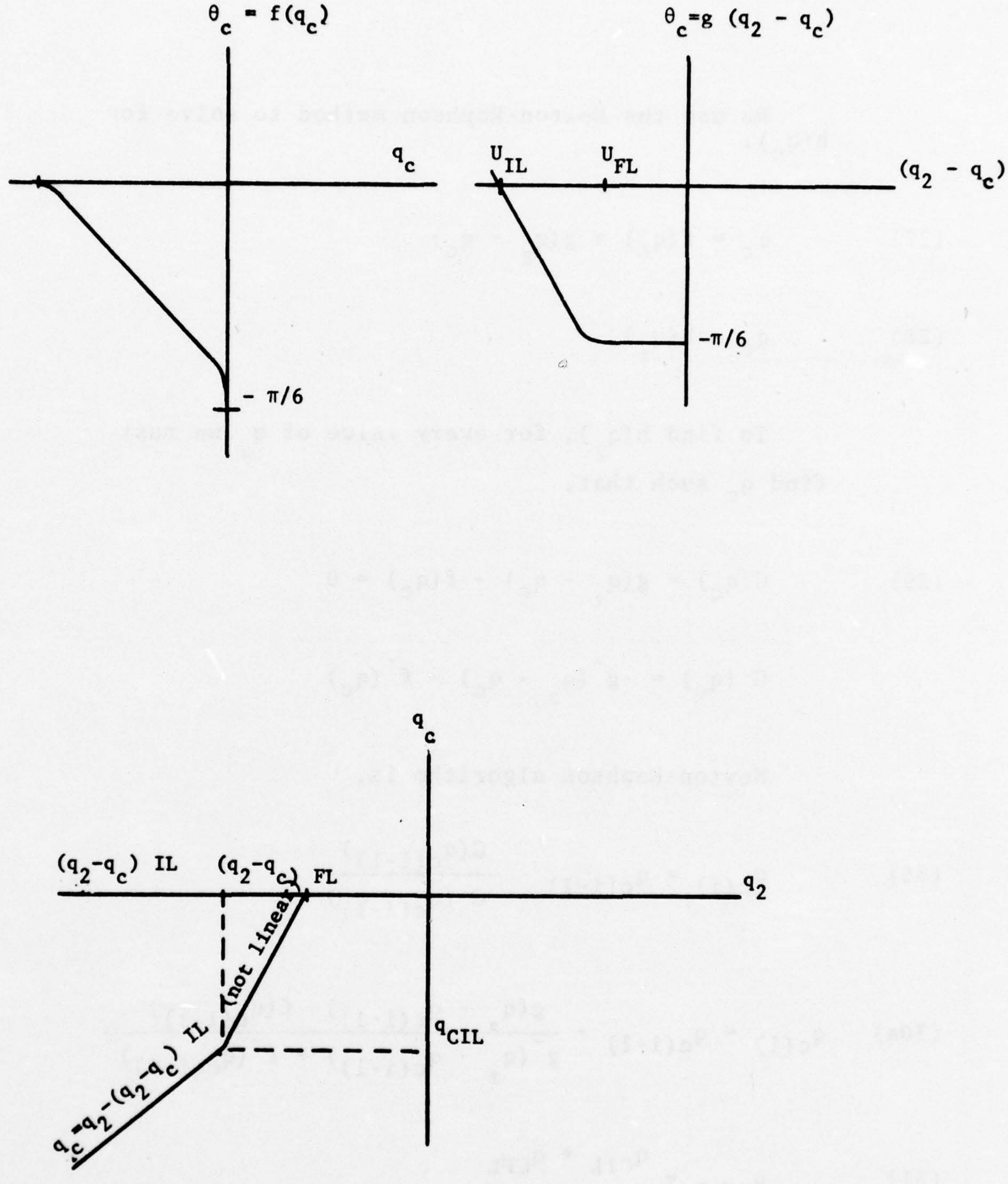
$$U_{IL} = (q_2 - q_c)_{IL} = -2.50 \text{ in}$$

(26) $$U_{qf} = (q_2 - q_c)_{qf} = U_{FL} - x_{qf}$$

From equations (19) and (25) we have,

(26a) $$U_{qf} = U_{FL} - \frac{Y_{\circ qf} \tan \theta_{\circ q}}{\frac{b^2_{qf}}{a^2_{qf}} + \tan^2 \theta_{\circ q}}$$

We have defined the two constraint functions $\theta_c = g\ (q_2 - q_c)$ and $\theta_c = f\ (q_c)$ which determines q_c *as a function of* q_2. Since these two constraint functions are not simple functions we cannot expect $q_c = h\ (q_2)$ to be simple. We first obtain a general understanding of $h\ (q_2)$ from $f\ (q_c)$ and $g\ (q_2 - q_c)$, then we show how to compute $h\ (q_2)$ along with the first and second derivatives which are required to analyze the dynamics of the system.


θc = f(qc)
qc
- π/6
θc=g (q2 - qc)
UIL
UFL
(q2 - qc)
-π/6
qc
(q2-qc) IL
(q2-qc) FL
q2
(not linear)
qCIL
qc=q2-(q2-qc) IL

We use the Newton-Raphson method to solve for $h(q_c)$.

(27) $$\theta_c = f(q_c) = g(q_2 - q_c)$$

(28) $$q_c = h(q_2)$$

To find $h(q_2)$, for every value of q_2 we must find q_c such that,

(29) $$G(q_c) = g(q_2 - q_c) - f(q_c) = 0$$

$$G'(q_c) = -g'(q_2 - q_c) - f'(q_c)$$

Newton-Raphson algorithm is,

(30) $$q_{c(i)} = q_{c(i-1)} - \frac{G(q_{c(i-1)})}{G\ (q_{c(i-1)})}$$

(30a) $$q_{c(i)} = q_{c(i-1)} + \frac{g(q_2 - q_{c(i-1)}) - f(q_{c(i-1)})}{g'(q_2 - q_{c(i-1)}) + f'(q_{c(i-1)})}$$

(31) $$q_{c(o)} = \frac{q_{CIL} + q_{CFL}}{2}$$

We iterate equation (30a) until,

$$[g - f] < \varepsilon_c$$

Nominal value:

(32) $$\varepsilon_c = 0.00001 \text{ rad}$$

We now obtain the first and second derivatives of $h(q_2)$.

(33) $$\frac{dq_c}{dq_2} = \frac{dq_c}{d\theta_c}\,\frac{d\theta_c}{dq_2} = h'(q_2)$$

Also from equation (27),

(34) $$\theta_c = f(q_c)$$

$$\frac{d\theta_c}{dq_c} = \frac{d}{dq_c}\,[f(q_c)] = f'(q_c)$$

(35) $$\frac{dq_c}{d\theta_c} = \frac{1}{f'(q_c)}$$

(36) $$\theta_c = g(q_2 - q_c)$$

let $x = q_2 - q_c$

$$\frac{dx}{dq_2} = 1 - \frac{dq_c}{dq_2}$$

$$\frac{d\theta_c}{dq_2} = \frac{d}{dq_2}[g(x)] = \frac{d}{dx}[g(x)]\frac{dx}{dq_2}$$

(37) $$\frac{d\theta_c}{dq_2} = g'(x)\left[1 - \frac{dq_c}{dq_2}\right]$$

From equations (33), (35), and (37),

(38) $$h'(q_2) = \frac{g'(q_2 - q_c)}{f'(q_c)}[1 - h'(q_2)]$$

(39) $$h'(q_2) = \frac{g'(q_2 - q_c)}{f'(q_c) + g'(q_2 - q_c)}$$

$$\frac{d}{dq_2}[h'(q_2)] = h''(q_2) = \frac{d}{dq_2}\left[\frac{g'(q_2 - q_c)}{f'(q_c) + g'(q_2 - q_c)}\right]$$

$$h''(q_2) = \frac{[f'(q_c) + g'(x)]\frac{d}{dq_2}[g'(x)] - g'(x)\frac{d}{dq_2}[f'(q_c)+g'(x)]}{[f'(q_c) + g'(x)]^2}$$

$$\frac{d}{dq_2}\,[f'(q_c)] = \frac{d}{dq_c}\,[f'(q_c)]\,\frac{dq_c}{dq_2} = f''(q_c)\,h'(q_2)$$

$$\frac{d}{dq_2}\,[g'(x)] = \frac{d}{dx}\,[g'(x)]\,\frac{dx}{dq_2} = g''(x)\;(1 - h'(q_2))$$

(40) $$h'' = \frac{g''\;(1 - h')}{(f' + g')} - \frac{g'\;[f''\,h' + g''\;(1 - h')]}{(f' + g')^2}$$

We are now going to obtain expressions for $f'(q_c)$, $f''(q_c)$, $g'(q_2 - q_c)$, and $g''(q_2 - q_c)$.

<u>For $q_c \geq q_{CFL}$:</u>

(41) $f(q_c)$

$f'(q_c)$ undefined

$f''(q_c)$

<u>For $q_{CF} < q_c < q_{CFL}$:</u>

$$f(q_c) = \delta_c + X_{CF}$$

$$X_{CF} = [a_{CF}{}^2 - \frac{a_{CF}{}^2}{b^2{}_{CF}}\;y^2{}_{CF}]^{\frac{1}{2}}$$

$$q_c = y_{CF} + (q_{CFL} - b_{CF})$$

(41a) $$f(q_c) = \delta_c + [a^2_{CF} - \frac{a^2_{CF}}{b^2_{CF}} (q_c - (q_{CFL} - b_{CF}))^2]^{\frac{1}{2}}$$

$$f'(q_c) = - \frac{\frac{a^2_{CF}}{b^2_{CF}} (q_c - q_{CFL} + b_{CF})}{f(q_c) - \delta_c}$$

$$f''(q_c) = \frac{- a^2_{CF}}{f(q_c) - \delta_c} [\frac{1}{b^2_{CF}} + (\frac{f'(q_c)}{a_{CF}})^2]$$

For <u>$q_{CI} < q_c \leq q_{CF}$</u> :

$$f(q_c) = (q_c - q_{CIL}) \tan \theta_{oc} + \varepsilon_{CI}$$

(41b) $$f'(q_c) = \tan \theta_{oc}$$

$$f''(q_c) = 0$$

For $q_{CIL} < q_c \leq q_{CI}$:

$$f(q_c) = \left[b^2{}_{CI} - \frac{b^2{}_{CI}}{a^2{}_{CI}} (q_c - q_{CIL})^2\right]^{\frac{1}{2}} - b_{CI}$$

(41c) $$f'(q_c) = - \left(\frac{b_{CI}}{a_{CI}}\right)^2 \left(\frac{q_c - q_{CIL}}{f(q_c) + b_{CI}}\right)$$

$$f''(q_c) = \frac{1}{f(q_c) + b_{CI}} \left[- \frac{b^2{}_{CI}}{a^2{}_{CI}} - f'(q_c)^2\right]$$

For $q_c \leq q_{CIL}$:

$$f(q_c) = 0$$

(41d) $$f'(q_c) = 0$$

$$f''(q_c) = 0$$

Let $U = q_2 - q_c$

For $U > (q_2 - q_c)_{FL}$:

$$g(u) = -\frac{\pi}{6}$$

(42) $$g'(u) = 0$$

$$g''(u) = 0$$

For $(q_2 - q_c)_{qf} < u \leq (q_2 - q_c)_{FL}$:

$$g(u) = b_{qf} - \frac{\pi}{6} - \left[b^2_{qf} - \left(\frac{b_{qf}}{a_{qf}}\right)^2 (u - u_{FL})^2\right]^{\frac{1}{2}}$$

(42a) $$g'(u) = \left(\frac{b_{qf}}{a_{qf}}\right)^2 \frac{(u - u_{FL})}{g(u) - b_{qf} + \frac{\pi}{6}}$$

$$g''(u) = \frac{-b^2_{qf}}{g(u) - b_{qf} + \frac{\pi}{6}} \left[\frac{1}{a^2_{qf}} + \left(\frac{g'(u)}{b_{qf}}\right)^2\right]$$

For $u \leq (q_2 - q_c)_{qf}$:

$$g(u) = (u - u_{IL}) \tan \theta_{oc}$$

(42b) $$g'(u) = \tan \theta_{oq}$$

$$g''(u) = 0$$

From equations (36) and (37),

$$\theta_c = g(q_2 - q_c)$$

(43) $$\frac{d\theta_c}{dq_2} = g'(q_2 - q_c)\ [1 - h'(q_2)]$$

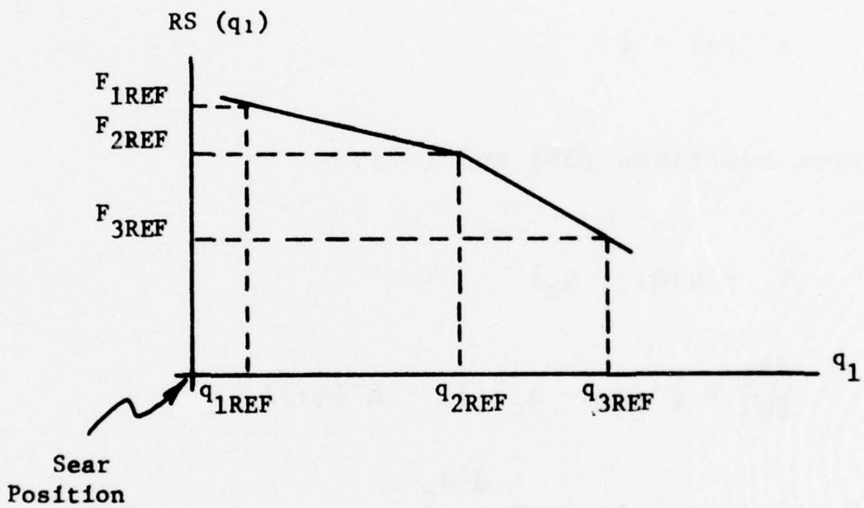
We also need to know $\dfrac{d^2\theta_c}{dq_2{}^2}$

(44) $$\frac{d^2\theta_c}{dq_2{}^2} = g''(u)\ [1 - h'(q_2)]^2 + g'(u)\ h''(q_2)$$

FORCE CALCULATIONS

1. Receiver recoil spring force:

$$FR_1 = RS(q_1)$$



$RS(q_1)$ - spring force as a function of the displacement relative to the turret.

For $q_1 \leq q_{2REF}$

$$RS(q_1) = \frac{F_{2REF} - F_{1REF}}{q_{2REF} - q_{1REF}} (q_1 - q_{2REF}) + F_{2REF}$$

For $q_1 > q_{2REF}$

$$RS(q_1) = \frac{F_{3REF} - F_{2REF}}{q_{3REF} - q_{2REF}} (q_1 - q_{3REF}) + F_{3REF}$$

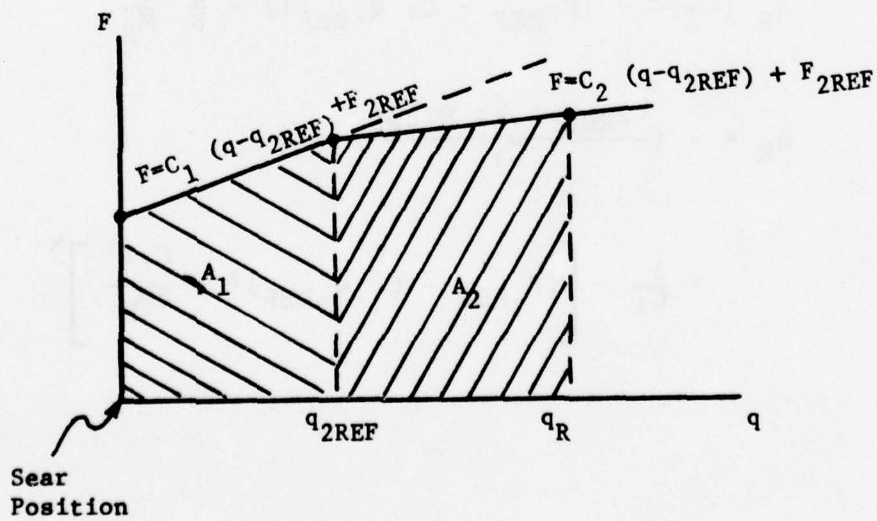
Nominal Values:

(q_{1REF}, F_{1REF}) = (0 in, 1266 lbs)

(q_{2REF}, F_{2REF}) = (2.85 in, 1224 lbs)

(q_{3REF}, F_{3REF}) = (5.7 in, 1136 lbs)

Calculation of recoil force from the force deflection curve.



$$A_1 + A_2 = \frac{1}{2} M_R \left(\frac{I}{2M_R}\right)^2$$

$$A_1 + A_2 = \frac{1}{8} \frac{I^2}{M_R}$$

If $A_1 > \frac{1}{8} \frac{I^2}{M_R}$, then $q_R < q_{2REF}$

otherwise,

$$q_R \geq q_{2REFF}$$

$$A_1 = q_{2REF} \left[F_{2REF} - \frac{C_1 q_{2REF}}{2}\right]$$

If $A_1 > \frac{1}{8} \frac{I^2}{M_R}$, then

$$q_R \left(\frac{C_1 q_R}{2} + (F_{2REF} - C_1 q_{2REF})\right) = \frac{1}{8} \frac{I^2}{M_R}$$

$$q_R = -\left(\frac{F_{2REF} - C_1 q_{2REF}}{C_1}\right)$$

$$+ \frac{1}{C_1} \left[(F_{2REF} - C_1 q_{2REF})^2 + \frac{C_1 I^2}{4M_R}\right]^{\frac{1}{2}}$$

If $A_1 \leq \frac{1}{8} \frac{I^2}{M_R}$, then

$$A_1 + (q_R - q_{2REF}) [F_{2REF} + \frac{1}{2} C_2 (q_R - q_{2REF})]$$

$$= \frac{1}{8} \frac{I^2}{M_R}$$

$$q_R = q_{2REF} + \frac{- F_{2REF} + \left[F_{2REF}^2 - 2C_2 (A_1 - \frac{1}{8} \frac{I^2}{M_R}) \right]^{\frac{1}{2}}}{C_2}$$

2. Receiver Friction:

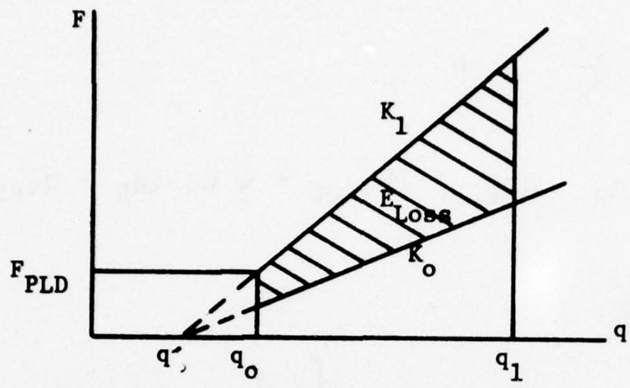
Assume that all surfaces moving relative to each other are well lubricated. We can assume under these conditions that dissipative forces acting on the receiver are dependent on only the velocity relative to the turret.

$$FR_2 = - C_{R_2} \dot{q}_1$$

where

C_{R_2} - Frictional coefficient (Nominal Value = 0)

3. The forward and rearward buffers are assumed to be linear springs, and the energy absorbed during compression and decompression is assumed to be due to "hysteresis." The energy loss is represented to be some percentage of the total potential (when mass has come to rest) of the spring.



F_{PLD}	Preload force
E_{LOSS}	Energy loss
% loss	Percent energy loss
q_o	Point mass contacts spring
q_1	Point mass comes to rest
q'	Point that force is zero

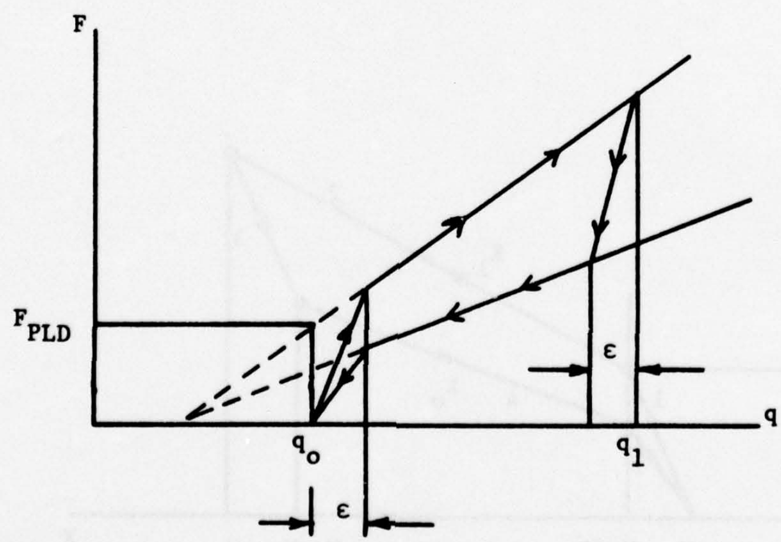
$$\frac{\%\ \text{loss}}{100} = \frac{\frac{1}{2} K_1 \left[(q_1 - q')^2 - (q_o - q')^2\right] - \frac{1}{2} K_o}{\frac{1}{2} K_1 \left[(q_1 - q')^2 - (q_o - q')^2\right]}$$

$$\frac{[(q_1 - q')^2 - (q_o - q')^2]}{\frac{1}{2} K_1 \left[(q_1 - q')^2 - q_o - q')^2\right]}$$

or

$$K_o = K_1 \left(1 - \frac{\%\ \text{loss}}{100}\right) \tag{3.1}$$

Due to computer limitations we must approximate "F" by a continuous function and where there is jump discontinuities we use a steep linear segment.

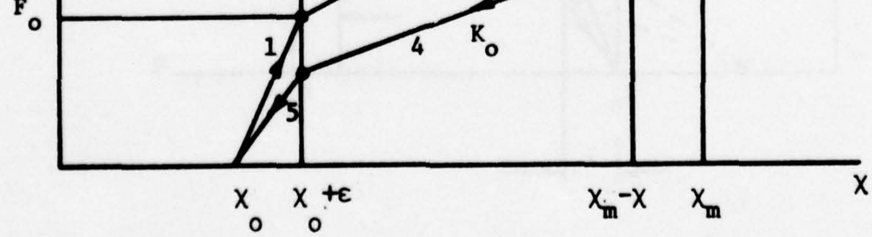


where ε - small

Nominal values for forward buffer:

$F_{PLD(F)}$	= 2600 lbs
$q_{o}(F)$	= 5.2 in
$q_{1}(F)$	= is the displacement (dependent on impact velocity, computed in program)
% $loss_{(F)}$	= 25%
$K_{1}(F)$	= 500 lbs/in
$K_{o}(F)$	= computed in program using equation (3.1)
$\epsilon(F)$	= 0.01 in.
FR_3	= - F

(This force computed by a subroutined, SPRG)



Branch 2:

(3.1.a) $$F = F_o + (x - x_o) K_1$$

Branch 1:

Using the two-point form for a straight line with the end points, (x_o, o) and $(x_o + \varepsilon, F_o + \varepsilon K_1)$

(3.1.b) $$F = \frac{(F_o + \varepsilon K_1)(x - x_o)}{\varepsilon}$$

Branch 5:

let $$G = 1 - \frac{\% \text{ loss}}{100}$$

(3.1.c) $$F = \frac{(F_o G + \varepsilon G K_1)(x - x_o)}{\varepsilon}$$

Branch 3:

End points = $(x_m, F_o + (x_m - x_o) K_1)$ and

$(x_m - \varepsilon, F_o G + (x_m - \varepsilon - x_o) G K_1)$

(3.1.d) $$F = F_o G + (x - x_m + \varepsilon) [K_1 (x_m - x_o) \frac{\% \text{ loss}}{100} + K_1 \varepsilon G$$

$$+ F_o \frac{\% \text{ loss}}{100}] / \varepsilon + (x_m - x_o - \varepsilon) K_1 G$$

Branch 4:

(3.1.e) $F = F_o G + (x - x_o) K_1 G$

4. Force from rearward buffer (see forward buffer analysis.

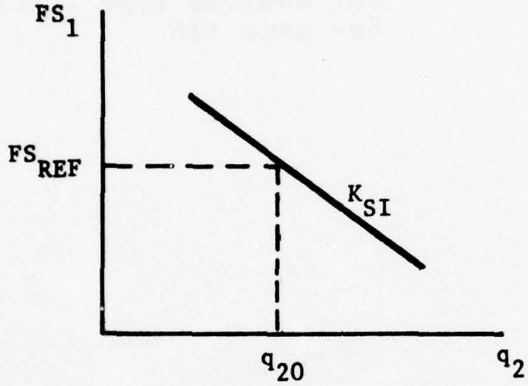
Nominal values for rearward buffer.

	$F_{PLD(R)}$	= 2600 lbs
	$q_o(R)$	= 0.2 in (- q is the displacement)
	$q_1(R)$	= dependent on impact velocity
	% $loss_{(R)}$	= 25%
	$K_1(R)$	= 500 lbs/in
	$K_o(R)$	= computed in program using equation (3.1)
	$\epsilon(R)$	= 0.01 in
	FR_4	= F (This force is computed by subroutine, SPRG)
(6)	FR_5	= $- FS_1$
	FR_6	= $- FS_2$
(7)	FR_7	= $- FS_3$
(8)	FR_8	= $- FS_4$
(9)	FR_9	= $- FS_5$
(10)	FR_{10}	= $- FS_6$
(11)	FR_{11}	= $- FS_7$
(12)	FR_{12}	= $- FC_1$

(13)	FR_{13}	$= - FC_2$
(14)	FR_{14}	$= - FC_3$
(15)	FR_{15}	= Piece-wise constant force acting on the receiver (accounts for coulomb type friction) See page 116

Forces Acting on the Slide:

1. Drive Spring Force, FS_1 :



q_{20}	- Reference position
FS_{REF}	- Force at reference position
K_{SI}	- Spring rate

Nominal values:

q_{20}	= Calculated (see page 115) - Full forward, slide, in equilibrium
FS_{REF}	= 290 lbs
K_{SI}	= 13.2 lbs/in

(Equal and opposite forces act on the receiver, see FR_5)

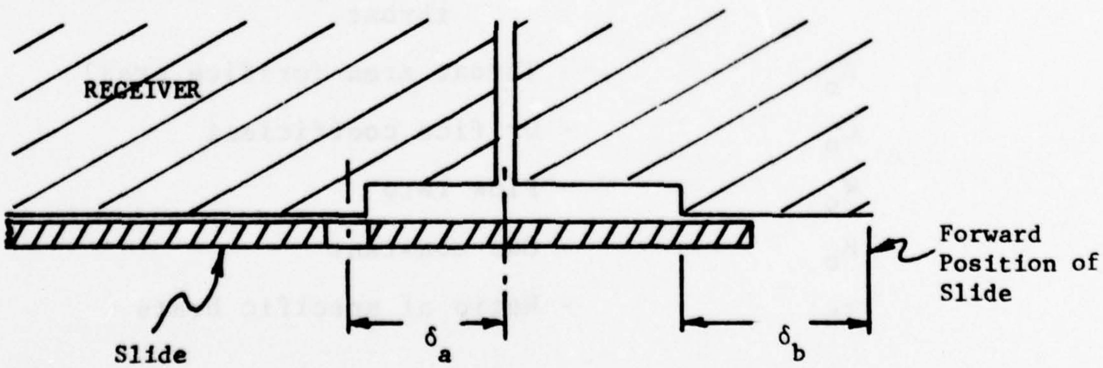
2. While the slide is in motion we assume the motion is opposed by a dissipative force dependent only on the velocity of the slide.

$$FS_2 = - C_{s2} \dot{q}_2$$

where C_{s_2} - is damping coefficient

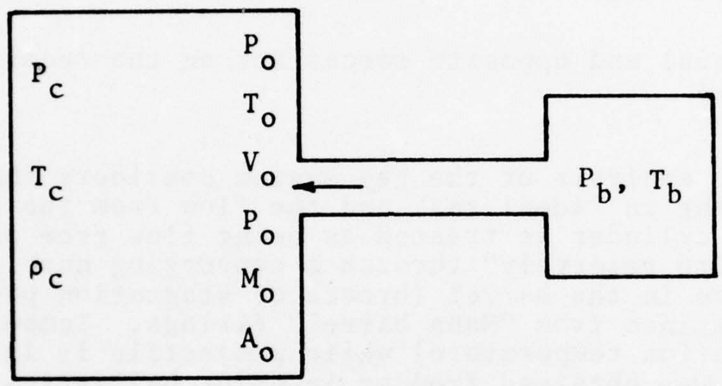
(Equal and opposite forces act on the receiver, see FR_6)

3. The analysis of the gas system considers the gas has being an "ideal gas" and the flow from the barrel to the cylinder is treated as being flow from an "infinite reservoir" through a converging nozzle. Pressure in the barrel (breech or stagnation pressure) was obtained from "Mann barrel" firings. Temperature (stagnation temperature) while projectile is in the barrel was obtained from an interior ballistics analysis. Once the projectile leaves the barrel, the temperature was computed from the pressure with the assumption the gas expands adiabatically. (This analysis is similar to analysis of Joseph H. Spurk, "Gas Flow in Gas Operated Weapons.")



δ_a - Distance slide travels from full forward position prior to gas being cut off to cylinder.

δ_b - Distance traveled (of slide) before flow can begin to be drained back to the barrel.



P_b, T_b - Pressure and temperature of the barrel gas.

P_c, T_c, ρ_c - Pressure, temperature and density of the cylinder gas.

P_o, T_o, ρ_o, V_o, M_o - Pressure, temperature, density, velocity, and mach number at throat.

A_o - Throat area (orifice area)

C_o - Orifice coefficient

W_o - Flow rate

R_b - Gas constant

γ_b - Ratio of specific heats

(3.1.1) $$\frac{W_o}{A_o} = C_o \left[\frac{\gamma_b}{R_b T_b}\right]^{1/2} P_o M_o \left[1 + \frac{\gamma_b - 1}{2} M_o^2\right]^{1/2}$$

(See "Compressible Fluid Flow," Shapino, page for equation (3.1.1).)

(3.1.2) $$\frac{P_b}{P_o} = \left(1 + \frac{\gamma_b - 1}{2} M_o^2\right)^{\frac{\gamma_b}{\gamma_b - 1}}$$

(3.1.3) $$\frac{T_b}{T_o} = 1 + \frac{\gamma_b - 1}{2} M_o^2$$

Energy equations for a deformable control volume (See "Foundation of Fluid Mechanics and Fluid Statics," Arthur G. Hansen, page 130.).

(3.2) $$\dot{Q} = \dot{W}_{SHAFT} + \dot{W}_{SHEAR} + \frac{d}{dt} \int_{V_{c.v.}} \rho \left(U + \frac{v^2}{2} + gz\right) dV$$

$$+ \int_{S_{c.v.}} \rho \left(h + \frac{v^2}{2} + gz\right) \bar{v}_r \cdot \hat{n}\, dA$$

$$+ \int_{S_{c.v.}} \rho\, \bar{v}_b \cdot \hat{n}\, dA$$

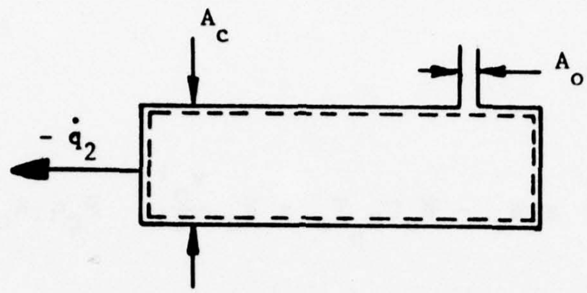
$\dot{Q}$	- Rate of heat transfer to system.
$\dot{W}_{SHAFT}$	- Shaft work being done by system.
$\dot{W}_{SHEAR}$	- Shear work being done by system.
ρ	- Density
U	- Internal energy per unit mass

v	- Velocity
h	- Enthalpy
gz	- Gravitational potential
$\hat{n}$	- Vector normal to control volume
$\bar{v}_r$	- Velocity vector relative to control volume
$\bar{v}_b$	- Velocity vector of deformable volume
dA	- Element of area
$\frac{d}{dt}$	- Time derivative
$\int_{V_{c.v.}}$	- Integral over control volume
$\int_{S_{c.v.}}$	- Integral over surface of control volume

We assume,

$$gz = \dot{Q} = \dot{W}_{SHAFT} = \dot{W}_{SHEAR} = 0 \tag{3.3}$$

Also, $\frac{v^2}{2} + gz$ is small compared to U inside the control volume (cylinder).



V_o	- Initial cylinder volume
V_c	- Cylinder volume
q	- Velocity of the slide
C_{v_b}	- Specific heat at constant volume of barrel gas
C_{p_b}	- Specific heat at constant pressure of barrel gas
e_c	- Internal energy per unit mass of the cylinder gas
P_c	- Cylinder pressure

The energy equation is given by:

$$\frac{d}{dt}(e_c \rho_c V_c) - W_o C_{p_b} T_o - W_o \frac{V_o^2}{2} - P_c \dot{q}_2 A_c = 0 \tag{3.4}$$

(NOTE: Ideal gas $h = C_p T$)

or

$$\frac{de_c}{dt} P_c V_c + e_c W_o - W_o C_{P_b} T_o - W_o \frac{v_o^2}{2} - P_c \dot{q}_2 A_c = 0 \tag{3.5}$$

Also,

$$\rho_c = (\rho_a V_o + \int W_o dt) / V_c \tag{3.6}$$

and

$$V_c = V_o - (q_2 - q_{REF}) A_c \tag{3.7}$$

Since the initial number of moles of gas in the cylinder is so small compared to the amount of gas that flows in, we can assume the gas constants (C_v, C_p and R) of the "initial gas" is the same as the barrel gas (gas constants in cylinder do not change).

$$R_c = C_{P_b} - C_{v_b} \tag{3.8}$$

$$P_c = R_c \rho_c T_c = (C_{P_b} - C_{v_b}) \rho_c T_c \tag{3.9}$$

$$e_c = C_{v_b} T_c \tag{3.10}$$

The mach number is computed according to the following relationships.

$$M_o = 1 \quad \text{if} \quad \frac{P_b}{P_c} \quad \left(\frac{\gamma_b + 1}{2}\right)^{\frac{\gamma_b}{\gamma_b - 1}} \qquad \text{(choked)} \tag{3.11}$$

If the flow is not choked (see "Shapiro", page 83)

$$M_o{}^2 = \left[\left(\frac{P_b}{P_c}\right)^{\frac{\gamma_b - 1}{\gamma_b}} - 1\right]\frac{2}{\gamma_b - 1} \qquad \text{if} \tag{3.12}$$

$$1 \leq \frac{P_b}{P_c} \leq \left(\frac{\gamma_b + 1}{2}\right)^{\frac{\gamma_b}{\gamma_b - 1}}$$

If

$$\frac{P_b}{P_c} < 1$$

then we have flow from the cylinder to the barrel. We interchange P_b and P_c , and apply equations (3.11) and (3.12). Also, we place a negative sign in front of (3.1.1) and replace P_b and T_b by P_c and T_c in equations (3.1.1), 3.1.2) and (3.1.3).

There is no flow if the slide is in a given region:

If

$\delta_b \leq -(q_2 - q_{REF}) \leq \delta_a$,

then

$$M_o = 0$$

$q_{REF} = \frac{FS_{REF}}{K_{1(SF)}}$ = Position of slide when it is in equilibrium position (forward position and not elevated) (See page 115)

Energy equation (3.5) along with equations of motion and the equations that describe the thermodynamic properties of the gas in the cylinder determines the system dynamics.

(3.13) $FS_3 = (P_c - P_a) A_c$

at

$$t = 0$$

(3.14) $T_c = T_a$ = atmospheric temperature

(3.15) $P_c = P_a$ = atmospheric pressure

(3.16) $\gamma_c = \gamma_c = P_a / (R_c T_a)$

Equal and opposite force acts on the receiver (See FR_7).

Nominal values:

$$A_c = \frac{\pi}{4}(0.588)^2 = 0.271 \text{ in}^2$$

$$\delta_a = 0.17 \text{ in}$$

$$\delta_b = 3.625 \text{ in}$$

$$A_o = 3\frac{\pi}{4}(0.12)^2 = 0.033929 \text{ in}^2$$

$$V_o = 11\frac{\pi}{4}(0.588)^2 + 1.62\frac{\pi}{4}(0.847)^2 + 0.375\ (1.218)^2$$

$$= 4.337 \text{ in}^3$$

$$C_{P_b} = 0.4326 \frac{\text{cal}}{\text{gram } ^\circ\text{C}} = 2.811 \text{ X } 10^6 \left(\frac{\text{in}}{\text{sec}}\right)^2 \frac{1}{^\circ\text{R}}$$

$$\gamma_b = 1.31$$

$$C_{v_b} = C_{P_b} / \gamma_b = 2.1459 \text{ X } 10^6 \left(\frac{\text{in}}{\text{sec}}\right)^2 \frac{1}{^\circ\text{R}}$$

$$P_a = 14.7 \text{ psi}$$

$$T_a = 530\ ^\circ\text{R}$$

$$C_o = 1$$

Barrel Pressure and Temperature Determination:

We use an experimental pressure curve. Temperature during "in-bore" time is obtained from interior ballistics calculations (see results from Darrel Thompson and Bill Leech). Once the projectile leaves the barrel, and assuming no heat loss and using the fact the gas does no work from this point, we see the adiabatic expansion law holds. We compute the temperature of the gas from the experimental pressure curve and the following.

$$\frac{T_b}{T_{bm}} = \left(\frac{P_b}{P_{bm}}\right)^{\frac{\gamma_b - 1}{\gamma_b}} \tag{3.17}$$

T_{bm} - Temperature of gas when projectile leaves barrel.

P_{bm} - Pressure of gas when projectile leaves barrel.

Gas begins to flow after the projectile reaches the port.

t_{PORT} = 0.00942 - time to reach port

Barrel Stagnation Pressure and Temperature

Time (Sec)	Pressure	Temperature (°R)
0	14.71	530
0.0001	23300	3741
0.00062	1600	3411
0.001042	11500	3323
0.00118	10510	EXIT
0.00268	5920	
0.00418	3740	
0.00568	2800	ADIABATIC
0.00718	1765	
0.01168	831	
0.01768	520	
0.02068	364	
0.050	14.71	

$P_{bm} = 11500$ $\qquad$ $T_{bm} = 3323$

4. Slide rear buffer force - FS_4

See pages 95 - 98 for the type of spring forces.

Nominal Values:

$F_{PLD(SR)}$ = 0.0 lbs (guessed)

$q_{o(SR)}$ = 9.817 in (-q is displacement)

$q_{1(SR)}$ = (Dependent on impact velocity)

% loss = 25%

$K_{1(SR)}$ = 64000 lbs/in (guessed)

$\epsilon_{(SR)}$ = 0.01 in

FS_4 = F

(Force computed by subroutine, SPRG.)

Equal and opposite force on the receiver (see FR_8).

5. To allow for time for the round to be fed into position in front of the chamber, the chamber becomes seared and sear is released when the receiver is some distance rearward in recoil. The buffering of the slide when it is restrained from moving forward during feeding is accomplished by part of the same buffering spring that stops the slide. (See pages 96 - 97 for type of spring.)

Nominal Values:

$F_{PLD(SS)}$ = 0.0 lbs

$q_{o}(SS)$ = 9.817 in (q_2 is the displacement)

$q_1(SS)$ = (dependent on impact velocity)

$K_1(SS)$ = 48000 lbs/in (guessed)

% loss = 25%

$\epsilon_{(SS)}$ = 0.01 in

FS_5 = -F

Equal and opposite force on receiver, FR_q.

FS_5 = F if,

$q_1 \geq q_{(1)RELEASE}$

$q_2 \geq q_{(2)LATCH}$ (during cycle)

$q_{(1)RELEASE}$ - distance receiver is rearward before slide is released.

$q_{(2)}$ LATCH - distance rearward the slide must be before it can become seared.

6. Forward Buffering Spring - FS_6

When the weapon is fully locked, this spring force restrains the slide from moving further forward relative to the receiver. To start the system integration, this spring must counter balance the drive spring force along with the weight component.

Nominal Values:

$F_{PLD(SF)}$ = 0.0 lbs

$q_{o}(SF)$ = 0.0 in (q_2 is the displacement)

$q_1(SF)$ = (dependent on impact velocity)

$K_1(SF)$ = 64000 lbs/in

$\varepsilon_{(SF)}$ = 0.01 in

% loss = 25%

FS_6 = -F

(Equal and opposite force acts on receiver, FR_{10})

<u>We now compute q_{20}</u>

$$q_{20} = (FS_{REF} - M_S\ g\ \text{Sin}\ \theta_o)/K_1(SF)$$

where

θ_o - Initial angle of elevation

FS_{REF} - See page 102

This value for q_{20} insures no unbalanced internal forces, initially.

NOTE: To calculate q_{20} we assumed the preload on the drive spring would not change. Since the forward buffer spring is much stiffer than the drive spring, then the penetration into the forward buffer will not

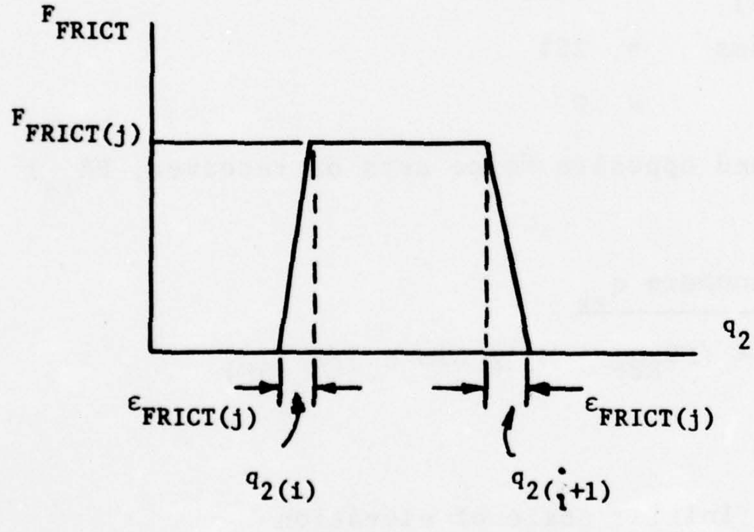
be significant enough to change the preload.

If we fire with slide rearward we assume the slide sear is released when receiver sear is released.

7. To allow us to look at the effect of forces due to stripping of rounds or coulomb friction, we define a piecewise constant force that acts between points $q_{2(i)}$ and $q_{2(i+1)}$ and opposes the motion of the slide.

If $\dot{q}_2 < 0$

$$FS_7 = F_{FRICT}$$



$i = 1, 2, \ldots, 2n$

$j = 1, 2, \ldots, n$

If $\dot{q}_2 > 0$

$FS_7 = - F_{FRICT}$

$i = 2n + 1, \ldots, 2n + 2m$

$j = n + 1, -.., m$

Associated with each force will be an equal and opposite force acting on either the receiver or turret. A control number is associated with each piecewise constant segment.

If 1 then,

$$FR_{11} = -FS_7$$

If 0 then,

$$FR_{11} = 0$$

(Nominal value of force is zero.)

(Total number of segments allowed is 3.)

$$n + m \leq 3$$

<u>Forces Acting on the Chamber</u>:

1. Rear buffer spring force - FC_1

   (See pages 96 - 97 for the type of spring force.)

Nominal Values:

$F_{PLD(CR)}$ = 1700 lbs

$q_o(CR)$ = 7.312 in ($-q_c$ is the displacement)

% loss = 25%

$K_{1(CR)}$ = 3400 lbs/in

$\varepsilon_{(CR)}$ = 0.01 in

FC_1 = F

(Force computed by subroutine, SPRG)

(Equal and opposite force acts on the receiver, FR_{12})

2. To account for viscous type friction we define

$$FC_2 = -C_c\dot{q}_c$$

Nominal Values:

C_c = 0

(Equal and opposite force acts on receiver, FR_{13})

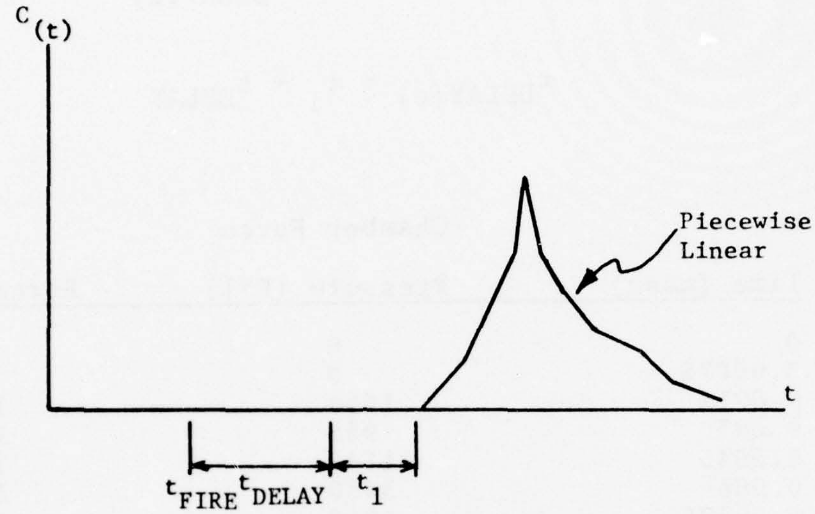
3. We define a piecewise constant force (similar to page 116) acting on the chamber (FC_3).

(An equal and opposite force acts on the receiver, FR_{14})

Nominal Value is zero

$$n + m \leq 3$$

4. The force from the propellant effectively acts on the receiver, since when the weapon is fired the chamber is locked to the receiver.



$c(t)$	-	Chamber pressure force
t_{FIRE}	-	Time of fire
t_{DELAY}	-	Ignition delay
FC	=	$-c(t)$

Nominal Values:

t_{FIRE}	=	(See control equation)
t_{DELAY}	=	0.0 msec
t_1	=	10.0 msec

Effective time delay - $t_{DELAY(e)}$

$$t_{DELAY(e)} = t_1 + t_{DELAY}$$

Chamber Force

Time (msec)	Pressure (PSI)	Force (lbs)
0	0	0
0.00075	0	0
0.0015	1450	1589
0.003	935	1024
0.0045	1140	1249
0.006	3430	3758
0.00675	6860	7516
0.0075	16310	17870
0.00767	24000	26294
0.00787	32000	35059
0.00802	40000	43824
0.00839	48310	52928
0.00895	40000	43824
0.00919	32000	35059
0.00942	24000	26294
0.00984	16000	17530
0.0105	10510	11515
0.0120	5920	6486
0.0135	3740	4098
0.015	2800	3068
0.0165	1765	1934
0.021	831	910
0.027	520	570
0.030	364	399
0.0305	0	0

$$\text{Force} = A * P = 1.0956\ P$$

Torsional Forces Acting on the Chamber

Other than constraint torsional forces on the chamber, there will also be frictional forces. We define a piecewise constant torsional force (similar to the translational force on page 116) acting on the chamber.

$$M_{c_1} = \text{piecewise constant}$$

(Nominal is zero; three piecewise constant segments)

Since we have no degree of freedom for roll, the equal and opposite torsional force applied to the chamber by the receiver does not need to be considered.

The forces acting on the receiver are given by:

$$F_{X_R} = \sum_i FR_i$$

Forces acting on the slide are given by:

$$F_{X_S} = \sum_j FS_j$$

Forces acting on the chamber are given by:

$$F_{X_C} = \sum_k FC_k$$

$$M_c = M_{c_1}$$

The only thing left to determine at this point is the control equations that determine when to fire.

Preliminary Matrix Relationships

Part A:

If A is a column matrix and a function of $\bar{q}$ (a column matrix)

$$\bar{q} = \begin{bmatrix} q_1 \\ q_2 \\ \cdot \\ \cdot \\ \cdot \\ q_n \end{bmatrix}$$

$$A = \begin{bmatrix} A_1 \ (\bar{q}) \\ A_2 \ (q) \\ \cdot \\ \cdot \\ \cdot \\ \\ An \ (\bar{q}) \end{bmatrix} \tag{1}$$

then from the chain rule for differentiation of a column matrix

$$\frac{dA}{dt} = \frac{\partial A}{\partial \bar{q}} \frac{d\bar{q}}{dt} \tag{1.1}$$

or we can write this in the form;

$$
(1.2) \quad \frac{dA}{dt} = \begin{bmatrix} \frac{\partial A_1(\bar{q})}{\partial q_1} & \frac{\partial A_1(\bar{q})}{\partial q_2} & \cdots & \frac{\partial A_1(\bar{q})}{\partial qn} \\ \vdots & & & \vdots \\ \frac{\partial An(\bar{q})}{\partial q_1} & \frac{\partial An(\bar{q})}{\partial q_2} & \cdots & \frac{\partial An(\bar{q})}{\partial qn} \end{bmatrix} \frac{d\bar{q}}{dt}
$$

If A is a function of t, $\bar{q}$ and $\dot{\bar{q}}$ then we can see that,

$$
(1.3) \quad \frac{dA(t,\bar{q},\dot{\bar{q}})}{dt} = \frac{\partial A}{\partial \dot{\bar{q}}} \frac{d^2\bar{q}}{dt^2} + \frac{\partial A}{\partial \bar{q}} \frac{d\bar{q}}{dt} + \frac{\partial A}{\partial t}
$$

Part B:

Lagrange's Equation:

$$
\frac{d}{dt}\left(\frac{\partial T}{\dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} + \frac{\partial P}{\partial \dot{q}_i} = Q_{q_i}
$$

$$
i = 1, \ldots n
$$

T - Kinetic Energy

V - Potential Energy

P - Dissipative Potential

Q_{q_i} - Generalized Forces (i = 1, 2, ... n)

q_i - Generalized Coordinates (i = 1, 2, ... n)

Using notation from Part A and letting

$$Q_{\bar{q}} = \begin{bmatrix} Q_{q_1} \\ Q_{q_2} \\ \cdot \\ \cdot \\ \cdot \\ Q_{q_n} \end{bmatrix}$$

Lagrange's equation (equation 2) becomes in matrix form:

$$(2.1) \qquad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\bar{q}}}\right)^T - \left(\frac{\partial T}{\partial \bar{q}}\right)^T + \left(\frac{\partial V}{\partial \bar{q}}\right)^T + \left(\frac{\partial P}{\partial \dot{\bar{q}}}\right)^T = Q_{\bar{q}}$$

To integrate the equations of motion, one needs to separate the second order derivatives from the first order terms. If we apply equation (1.3) to the term

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\bar{q}}}\right)^T$$

in equation (2.1) (Note: Assuming that kinetic energy is a function of t, $\bar{q}$ and $\dot{\bar{q}}$) Larange's equation becomes:

$$\frac{\partial}{\partial \dot{\bar{q}}} \left(\frac{\partial T}{\partial \dot{\bar{q}}}\right)^T \ddot{\bar{q}} + \frac{\partial}{\partial \bar{q}} \left(\frac{\partial T}{\partial \dot{\bar{q}}}\right)^T \dot{\bar{q}} + \frac{\partial}{\partial t} \left(\frac{\partial T}{\partial \dot{\bar{q}}}\right)^T - \left(\frac{\partial T}{\partial \bar{q}}\right)^T$$

$$+ \left(\frac{\partial V}{\partial \bar{q}}\right)^T + \left(\frac{\partial P}{\partial \dot{\bar{q}}}\right)^T = Q_{\bar{q}} \tag{2.2}$$

With this form of the equation, if we are able to determine T, V, and P as functions of the generalized coordinates, we can use an "algebraic computer language" to generate the equations of motion. The terms

$$\frac{\partial}{\partial \dot{\bar{q}}} \left(\frac{\partial T}{\partial \dot{\bar{q}}}\right)^T, \quad \frac{\partial}{\partial \bar{q}} \left(\frac{\partial T}{\partial \dot{\bar{q}}}\right)^T, \quad \frac{\partial}{\partial t} \left(\frac{\partial T}{\partial \dot{\bar{q}}}\right)^T, \quad \left(\frac{\partial T}{\partial \bar{q}}\right)^T, \quad \left(\frac{\partial V}{\partial \bar{q}}\right)^T, \text{ and}$$

$$\left(\frac{\partial P}{\partial \dot{\bar{q}}}\right)^T$$

are easily generated, punched on cards, or saved in some other form. These expressions can be used in an integration routine.

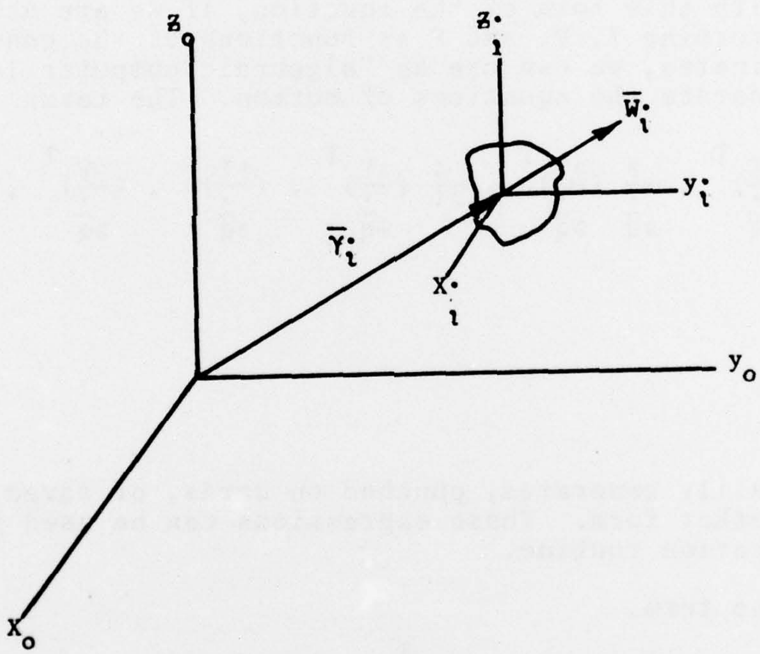
The term,

$$\frac{\partial}{\partial \dot{\bar{q}}} \left(\frac{\partial T}{\partial \dot{\bar{q}}}\right)^T$$

can be considered a generalized mass matrix. In order to compute the second order derivatives, the inverse of this term must exist. We denote the mass matrix by:

$$M = \frac{\partial}{\partial \dot{\bar{q}}} \left(\frac{\partial T}{\partial \dot{\bar{q}}}\right)^T$$

We now specialize these equations to a system of rigid bodies. We show that if the center of gravity, C.G., is determined relative to the inertial reference frame in terms of the generalized coordinates and the angular velocity of each body relative to a fixed body axis and in terms of the generalized coordinates is determined, then we show how to use the computer to operate on these expressions to generate the equations of motion.



$X_o Y_o Z_o$	Inertial coordinate system
$X_i Y_i Z_i$	Body axis of the "i-th" mass (Origin at the C.G. of the "i-th" mass).
$\overline{r}_i$	Vector, locating C.G. of the "i-th" mass relative to the inertial reference frame.
$\overline{W}_i$	Angular velocity of the "i-th" mass relative to the body axis.

The kinetic energy of the "i-th" mass can be written as:

$$T_i = \frac{1}{2} M_i \overline{r}_i^T \overline{r}_i + \frac{1}{2} \overline{W}_i^T I_i \overline{W}_i$$

where

M_i	Mass of the "i-th" body
I_i	Inertial tensor of "i-th" mass

$$I_i = \begin{bmatrix} I_{x_i x_i} & I_{x_i y_i} & I_{x_i z_i} \\ I_{y_i x_i} & I_{y_i y_i} & I_{y_i z_i} \\ I_{z_i x_i} & I_{z_i y_i} & I_{z_i z_i} \end{bmatrix}$$

(Note: $I_i = I_i^T$)

Identify $\bar{r}_i$ and $\bar{W}_i$ as being column matrices:

$$\bar{r}_i = \begin{bmatrix} r_{x_i} \\ r_{y_i} \\ r_{z_i} \end{bmatrix} \qquad \bar{W}_i = \begin{bmatrix} W_{x_i} \\ W_{y_i} \\ W_{z_i} \end{bmatrix}$$

also $\bar{r}_i$ and $\bar{W}_i$ are functions of $\bar{q}$, $\dot{\bar{q}}$ and t:

$$\bar{r} = \bar{r} \quad (\bar{q},\ t)$$

$$\bar{W} = \bar{W} \quad (\bar{q},\ \dot{\bar{q}},\ t)$$

The total kinetic energy is given by:

$$T = \sum_i T_i = \sum_i \left(\frac{1}{2} M_i \dot{\bar{r}}_i^{T} \dot{\bar{r}}_i + \frac{1}{2} \bar{W}_i^{T} I_i W_i\right) \tag{2.3}$$

Also, $\dot{\bar{r}}_i$ is given by:

$$\dot{\bar{r}}_i = \frac{\partial \dot{\bar{r}}_i}{\partial \bar{q}} \dot{\bar{q}} + \frac{\partial r_i}{\partial t}$$

We can now rewrite equation (2.2) as:

$$(2.4)\qquad \sum_i \frac{\partial}{\partial \dot{\bar{q}}}\left(\frac{\partial T_i}{\partial \dot{\bar{q}}}\right)^T \ddot{\bar{q}} + \sum_i \frac{\partial}{\partial \bar{q}}\left(\frac{\partial T_i}{\partial \dot{\bar{q}}}\right)^T \dot{\bar{q}} + \sum_i \frac{\partial}{\partial t}\left(\frac{\partial T_i}{\partial \dot{\bar{q}}}\right)^T$$

$$- \sum_i \left(\frac{\partial T_i}{\partial \bar{q}}\right)^T + \left(\frac{\partial V}{\partial \bar{q}}\right)^T + \left(\frac{\partial P}{\partial \dot{\bar{q}}}\right)^T = Q_{\bar{q}}$$

The generalized mass matrix is given by:

$$\sum_i M_i = \sum_i \frac{\partial}{\partial \dot{\bar{q}}}\left(\frac{\partial T_i}{\partial \dot{\bar{q}}}\right)^T$$

Lower order terms involving T_i are given by:

$$\sum_i C_i = \sum_i \left[\frac{\partial}{\partial \bar{q}}\left(\frac{\partial T_i}{\partial \dot{\bar{q}}}\right)^T \dot{\bar{q}} + \frac{\partial}{\partial t}\left(\frac{\partial T_i}{\partial \dot{\bar{q}}}\right)^T - \left(\frac{\partial T_i}{\partial \bar{q}}\right)^T\right]$$

The mass matrix of the system is just the sum of mass matrices from each body. Also, the first order terms are given by the sum of the lower order terms.

We can derive equations of motion by looking at a particular mass and determine $\bar{r}_i$ and $\bar{w}_i$; use the computer to determine M_i and C_i.

(M_i - mass matrix; C_i - column matrix of lower order terms.)

These expressions are printed out on cards or some other convenient form and used in an integration routine. During integration of equations of motion by numerical methods we compute

(2.5) $$\ddot{\bar{q}} = M^{-1} \left(- \frac{\partial P}{\partial \dot{\bar{q}}} - \frac{\partial V}{\partial \bar{q}} - \sum_i C_i + Q_{\bar{q}}\right) = f\ (\bar{q}, \dot{\bar{q}}, t)$$

If we use a routine that solves a system of the form;

$$\dot{\bar{y}} = f_1\ (x, \bar{y})$$

(2.6) $$\bar{y}\ (x_o) = \bar{y}_o \text{ (Initial conditions)}$$

then we need to transform (2.5) into that form.

Let

(2.7) $$\bar{q}_1 = \bar{q}$$

$$\bar{q}_2 = \dot{\bar{q}}$$

(2.8) $$\bar{y} = \begin{bmatrix} \bar{q}_1 \\ \\ \bar{q}_2 \end{bmatrix}$$

$$(2.9) \qquad \dot{\bar{y}} = \begin{bmatrix} \dot{\bar{q}}_1 \\ \dot{\bar{q}}_2 \end{bmatrix} = \begin{bmatrix} \bar{q}_2 \\ f\,(\bar{q}_1,\ \bar{q}_2,\ t) \end{bmatrix} = f_1\,(\bar{y},\ t)$$

$$(2.10) \qquad \bar{y}_o = \begin{bmatrix} q_o \\ \dot{\bar{q}}_o \end{bmatrix} \qquad \text{(Initial conditions)}$$

So far nothing has been said about generalized forces. If

$$\bar{r}_{f_p}$$

is the point of application of a force

$$\bar{F}_p = \begin{bmatrix} F_{x_p} \\ F_{y_p} \\ F_{z_p} \end{bmatrix}$$

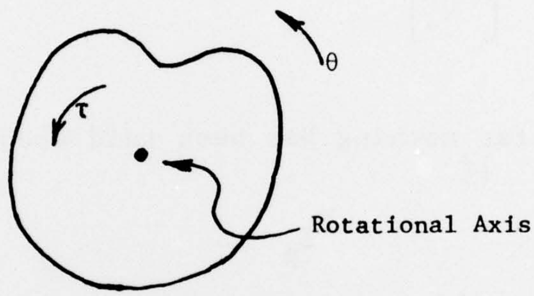
then,

$$(2.11) \qquad Q_{\bar{q}} = \sum_p \frac{\partial \bar{r}_{f_p}}{\partial \bar{q}} \bar{F}_p$$

(Note: From literature $Q_{q_i} = \frac{\partial \bar{r}_{f_p}}{\partial q_i} \bar{F}_p$)

We can use the computer to derive the equations that determine the generalized forces. Read in $\bar{r}_{f_p}$ and $\bar{F}_p$, and compute $Q_{\bar{q}}$.

We now obtain the expression for the generalized force when a moment is applied in the direction of a given axis.



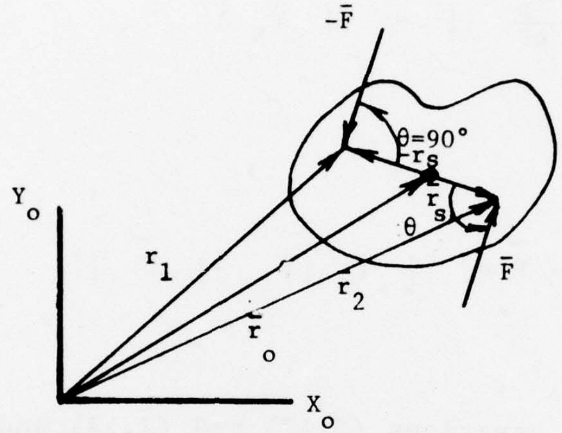
We will show that,

$$Q_{q_i} = \tau \frac{\partial \theta}{\partial q_i}$$

Replace, torque τ by the couple (see diagram below).

$\tau = 2 \,||\, \bar{F} \,||\;\; ||\, \bar{r}_s \,||\, \text{Sin}\ \theta$

$\tau = 2 \,||\, \bar{F} \,||\, ||\, \bar{r}_s \,||$



$$\bar{r}_1 = \bar{r}_o - \bar{r}_s$$

$$\bar{r}_2 = \bar{r}_o + \bar{r}_s$$

The generalized force from the couple is

$$Q_{q_i} = \frac{\partial \bar{r}_1}{\partial q_i} \cdot \bar{F} + \frac{\partial r_2}{\partial q_i} \cdot \bar{F} \tag{2.12}$$

$$= 2 \frac{\partial r_s}{\partial q_i} \cdot \bar{F}$$

$$= 2 \left(\frac{\partial \bar{r}_s}{\partial \theta} \cdot \bar{F}\right) \frac{\partial \theta}{\partial q_i}$$

Since $\bar{F}$ is $\| \frac{\partial \bar{r}_s}{\partial \theta}$ and,

$$(2.13) \qquad \left\| \frac{\partial \bar{r}_s}{\partial \theta} \right\| = \| \bar{r}_s \|$$

then

$$(2.14) \qquad \frac{\partial \bar{r}_s}{\partial \theta} \cdot \bar{F} = \| \bar{r}_s \| \quad \| \bar{F} \|$$

From equations (2.12) and (2.14) and the fact $\tau = 2 \quad \| \bar{F} \| \, \| \bar{r}_s \|$

$$(2.15) \qquad Q_{q_i} = \tau \frac{\partial \theta}{\partial q_i}$$

θ is a function of the generalized coordinates and the computer can be used to generate (2.15). In vector notation (2.15) is written as:

$$(2.16) \qquad Q_{\bar{q}} = \tau \frac{\partial \theta}{\partial \bar{q}}$$